SURVIVAL MODELS (30 hours)

1. Survival models and the life table
- A simple model of survival (lifetime distribution, rates of mortality, probabilities of death and survival, force of mortality, central rate of mortality)
- Complete and curtate expectation of life
- Life tables
- Life table functions at non-integer ages (uniform distribution of deaths assumption)
- The general pattern of mortality
- Simple laws of mortality (Gompertz' and Makeham's laws)

2. Estimating the lifetime distribution
- Questions of inference (parametric and non-parametric models)
- Censoring mechanisms (right, left, interval, random, non-informative, type I and II censoring)
- The Kaplan-Meier estimator (Greenwood's formula for variance of the estimator)
- The Nelson-Aalen estimate (variance of the estimator)
- Parametric models (covariates)
- The Cox model (semi-parametric model, regression parameters, partial likelihood)

3. The two-state Markov model
- Basic assumptions
- Transition probabilities (Kolmogorov forward equations)
- Statistics
- The maximum likelihood estimator for transition intensity

4. The general Markov model
- Assumptions
- Kolmogorov equations
- Statistics

5. Binomial and Poisson models
- Binomial models (assumptions, distributions of statistics, likelihood of parameters)
- Assumptions about mortality (uniform, Balducci, constant force of mortality)
- The actuarial estimate
- Poisson models (assumptions, distributions of statistics, maximum likelihood estimator of the (constant) intensity)
- Comparison of multiple-state, Binomial and Poisson models

6. Graduation and statistical tests
- Comparison with another experience (crude estimates with standard tables)
- Graduation of crude estimates

- Desirable features of a graduation
- Testing the smoothness of a graduation (definition of smoothness and reasons)
- Statistical tests of a mortality experience (the hypothesis of consistency of crude estimates with a standard table, standardised deviations, χ^2-test, standardised deviations test, signs test, cumulative deviations test, grouping of signs test, serial correlation test)

7. Methods of graduation
- Graduation by parametric formula (Gompertz, Makeham)
- Graduation by reference to a standard table
- Graphical graduation
- Comparison of different models
- Statistical tests of graduation
- The effect of duplicate policies

8. Exposed to risk
- Connection between estimation of transition intensities and exposed to risk (central and initial exposed to risk)
- Homogeneity
- The principle of correspondence
- Exact calculation of the central exposed to risk
- Census approximations to central exposed to risk (trapezium approximation)
- Different definitions of age (last, next, nearest birthday)
- Calendar and policy year rate intervals

9. Heterogeneity within population and selection
- Factors affecting mortality and kinds of selection
- Transition intensities which depend on both age and duration
- Estimating the transition intensities
- Displaying the estimated results
- Constructing select and ultimate life tables
- Using tabulated select life table functions

10. The evaluation of assurances and annuities (expected present value (EPV) and variance of a sum assured, actuarial notations and examples)
- Life insurance contracts
- Whole of life assurance contracts
- Term assurance contracts
- Pure endowment assurance contracts
- Endowment assurance contracts
- Life annuity contracts
- Whole of life immediate annuity
- Whole of life annuity-due
- Temporary immediate annuity

- Temporary annuity-due
- Relationships among EPVs of simple life insurance benefits
- Deferred annuities and assurances
- Continuous annuities and assurances payable at the moment of death
- Commutation functions
- Retrospective accumulations

11. Premiums and reserves
- Equations of value
- Premiums (net premium, office premium)
- Prospective policy value
- Reserves and retrospective policy values
- Conditions for equality of prospective and retrospective policy values
- Net premium policy values
- Recursive calculation of policy values (conditions for, applications)
- Mortality profit (death strain at risk (DSAR), expected death strain (EDS), actual death strain (ADS))
- Theile's differential equation
- Calculating net premiums and net premium policy values using select mortality tables

References:

1. H.U. Gerber, *Life Insurance Mathematics,* Springer-Verlag Berlin Heidelberg and Swiss Association of Actuaries Zürich, 1990.
2. Chin Long Chiang, *Introduction to Stochastic Processes in Biostatistics*, Wiley, 1968.
3. N.L Bowers *et al*., *Actuarial Mathematics,* 2nd edition, Society of Actuaries, 1997.
4. S. Haberman, E. Pitacco, *Actuarial Models for Disability Insurance*, Chapman & Hall, 1999.
5. E. Marubini, M.G. Valsecci, M. Emmerson, *Analysing Survival Data from Clinical Trials and Observational Studies,* Wiley, 1995.
6. B. Benjamin, J.H. Pollard, *The Analysis of Mortality and Other Actuarial Statistics,* 3rd edition, Institute of Actuaries and Faculty of Actuaries, 1993.
7. R.C. Elandt-Johnson, N.L. Johnson, *Survival Models and Data Analysis*, Wiley, 1980.
8. *Subject*104: *Survival Models*, *Core Reading 2000*, Faculty and Institute of Actuaries
9. *Subject*105: *Actuarial Mathematics 1*, *Core Reading 2000*, Faculty and Institute of Actuaries