STOCHASTIC MODELLING (30 hours)

1. **Model structure and the principles of actuarial modelling**
   - models: how and why to use them
   - benefits and limitation of modelling
   - deterministic and stochastic models
2. **Stochastic processes: definition and classification**
   - general definition of stochastic process
   - classification of stochastic processes depending on time parameter and state space
   - classification of stochastic processes depending on interdependence: stationarity, Markov property, martingales
   - examples of stochastic processes: white noise, general random walk, moving averages, Poisson process, compound Poisson process, Brownian motion
   - martingales and stopping times
3. **Markov chains**
   - definition and basic properties of Markov chains
   - time homogeneous Markov chains and transition matrix
   - the Chapman-Kolmogorov equations
   - examples: no claims discount systems
   - stationary and limiting distribution of Markov chain
   - application to no claims discount systems
4. **Markov jump procesess**
   - definition and basic properties of Markov jump processes
   - the Kolmogorov equations
   - Poisson proces
   - structure of Markov jump processes: time homogeneous and inhomogeneous case
   - applications in insurance: sickness and death with and without time dependence, accident proneness
   - numerical methods
5. **Time series analysis**
   - properties of univariate time series
   - stationary random sequences
   - filtering of time series
   - backward shift operator and finite difference operator
   - basic linear models of time series: AR, MA, ARMA and ARIMA models
   - the frequency domain analysis of time series
   - multivariate autoregressive models
   - integrated and cointegrated time series
   - the Box-Jenkins approach to identification, estimation and diagnosis of time series
   - special non-stationary and non-linear time series models
   - applications of linear time series models
6. **Brownian motion and diffusions**

- introduction to Brownian motion
- Brownian motion and random walks
- application in insurance: ruin probability
- diffusions
- stochastic calculus: Ito's integral and stochastic differential equations
- Levy processes

7. Introduction to Monte Carlo simulation of stochastic processes

- generating pseudo-random numbers by computers, multiplicative algorithm
- simulation of random variables: inverse transform method, acceptance - rejection method, Box-Müller algorithm, polar algorithm
- simulation of a sequence of correlated normal random variables
- comparison of pseudo-random and random numbers

8. Process of actuarial modelling

- model proposal
- model valuation
- definining scenario
- sensitivity testing
- model limitations
- communication of the results

Literature:

1. T.Rolski, H.Schmidli, V.Schmidt, J.Teugels (1998) *Stochastic Processes for Insurance and Finance,* Wiley
2. S.I.Resnick (1992), *Adventures in Stochastic Processes.* Birkhäuser, Basel
3. G.R.Grimmett, D.R.Strizaker (1992), *Probability and Random Processes.* Clarendon Press, Oxford
4. P.J.Brockwell, R.A.Davis (1987), *Time Series: Theory and Methods.* Springer, New York
5. G.E.P.Box, G.M.Jenkins (1976), *Time Series Analysis: Forecasting and Control.* Holden Day, San Francisco
6. D.Lamberton, B.Lapeyre (1996), *Introduction to Stochastic Calculus Applied to Finance.* Chapmann&Hall
7. Faculty & Institute of Actuaries, *Core Reading Subject 103*