Health Insurance: Products

and

Basic Actuarial Models

Ermanno Pitacco

ermanno.pitacco@deams.units.it

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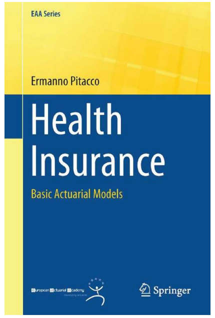
Agenda

- Introduction & motivation
- The need for health insurance
- Health insurance products
- Introduction to actuarial aspects
- Actuarial models for sickness insurance
- Actuarial models for disability annuities
- Long-term care insurance premiums: sensitivity analysis

Basic references:



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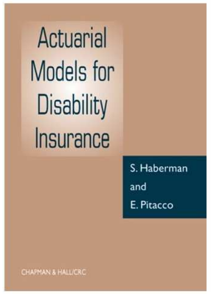

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INTRODUCTION & MOTIVATION

Look at the terminology adopted in the insurance practice

You find, for example:

- *accident* and *sickness* insurance $\Rightarrow$ the “cause” is referred to
- *disability* insurance $\Rightarrow$ the “physical effect” is referred to
- *loss-of-income* insurance $\Rightarrow$ the “financial effect” is referred to
- *income protection* insurance $\Rightarrow$ the “purpose” (of the insurance policy) is referred to
- *long-term care* insurance $\Rightarrow$ the “physical need” is referred to

Really, as regards the language, a “babel” situation !

And, as regards the language:

> "Die Grenzen meiner Sprache sind die Grenzen meiner Welt",
> Ludwig J. J. Wittgenstein, *Tractatus Logico-Philosophicus* (prop. 5.6), 1921

Hence, albeit accepting a well-established language, a primary concern is: to define an "insurance area" in which we can place all the insurance products which provide benefits related to individual health conditions

Main difficulty: in any given market (country), relation between public health care system and social security on the one hand and private health insurance and related products on the other

(My) definition:

> *In a broad sense, the expression "health insurance" denotes a large set of insurance products which provide benefits in the case of need arising from either accident or illness, and leading to loss of income (partial or total, permanent or non-permanent), and/or expenses (hospitalization, medical and surgery expenses, nursery, rehabilitation, etc.)*
>
> *Health insurance, in its turn, belongs to the area of the "insurances of the person"*

THE NEED FOR HEALTH INSURANCE

INDIVIDUAL CASH-FLOWS

Refer to an individual, starting his/her working period

Focus on the following cash-flows

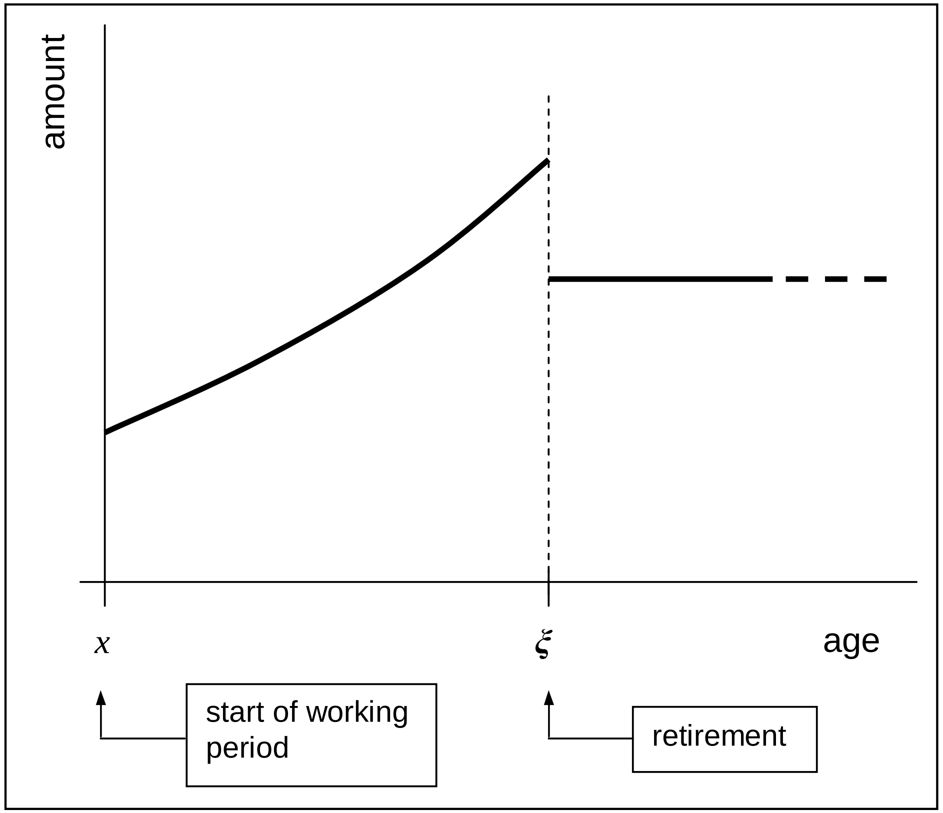
- inflows:
  - ▷ earned income (wage / salary)
  - ▷ pension (+ possible life annuities)
- outflows: health-related costs
  - ▷ medical expenses (medicines, hospitalization, surgery, etc.)
  - ▷ expenses related to long-term care

Age

x: start of the working period

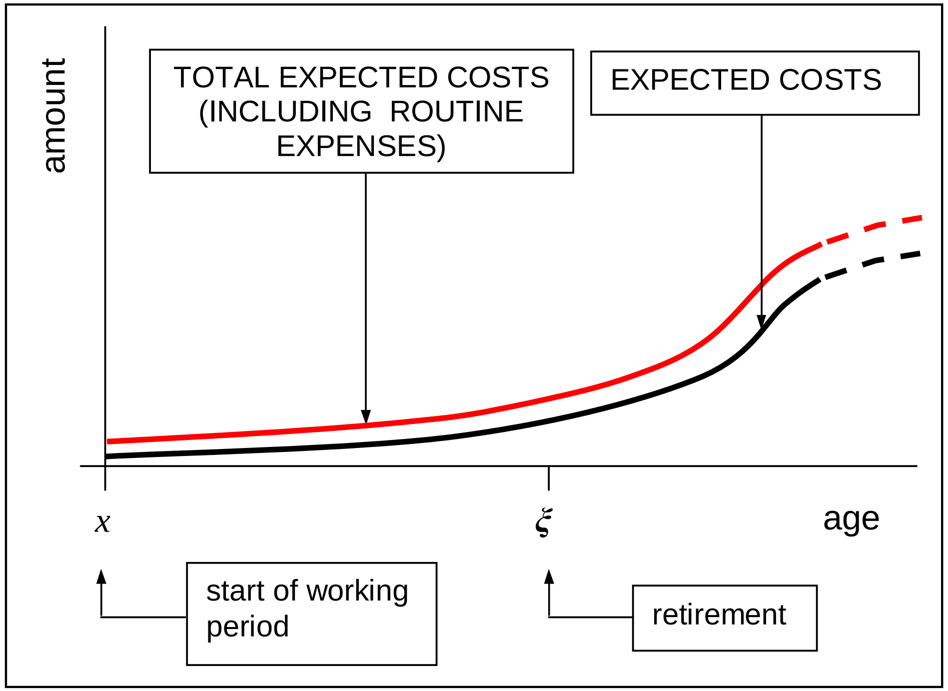
ξ: retirement

Income profile



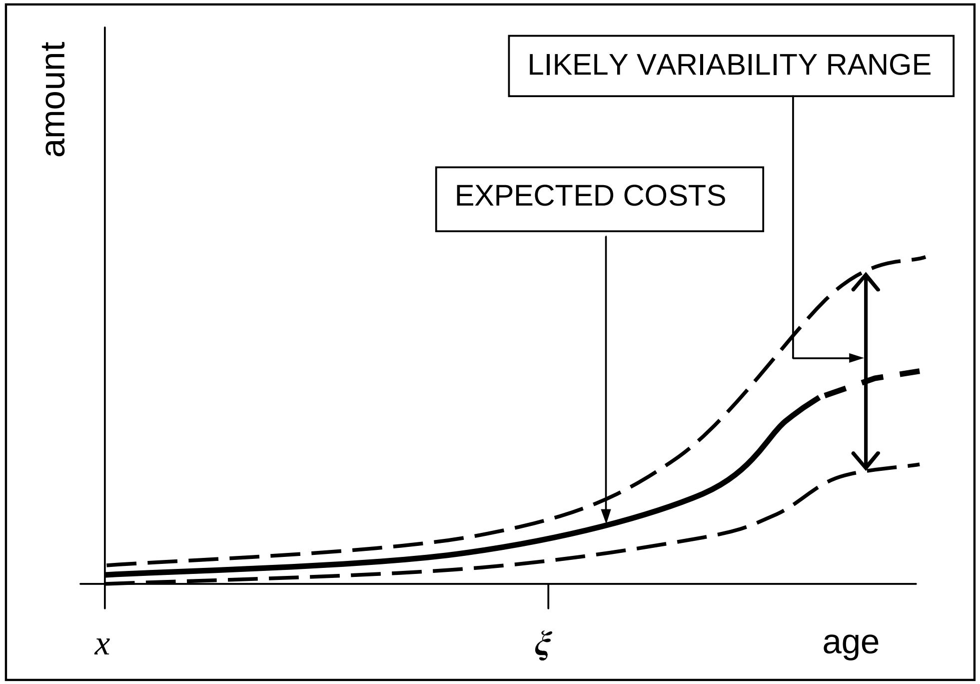
The income profile: an example

Time profile of health-related costs



Health-related expected costs

The need for health insurance *(cont'd)*

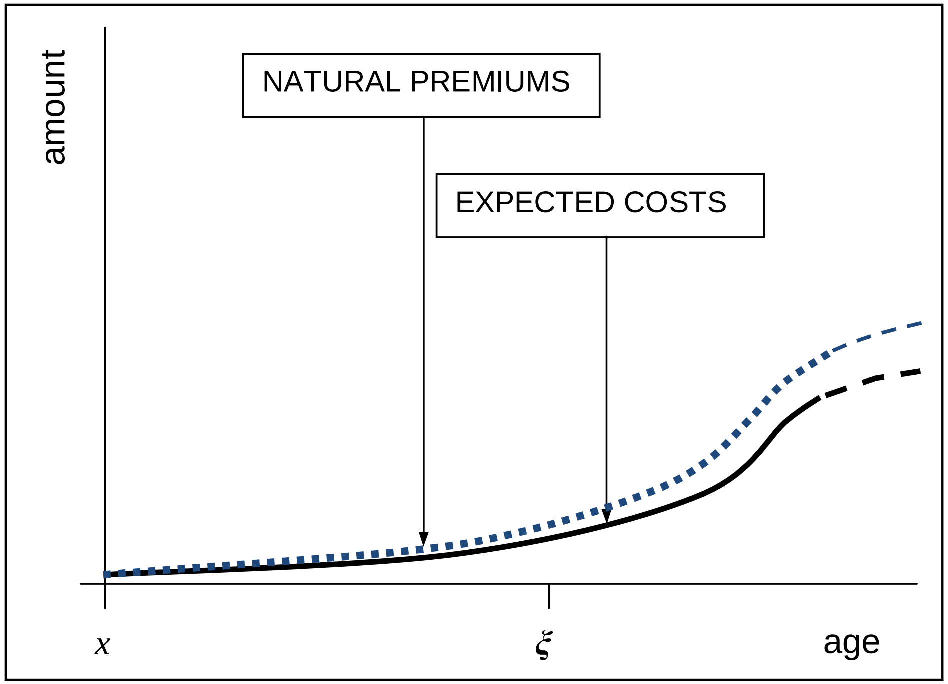


Health-related costs: expected value and variability

Following Figures: health-related costs (excluding routine expenses) financed via insurance cover

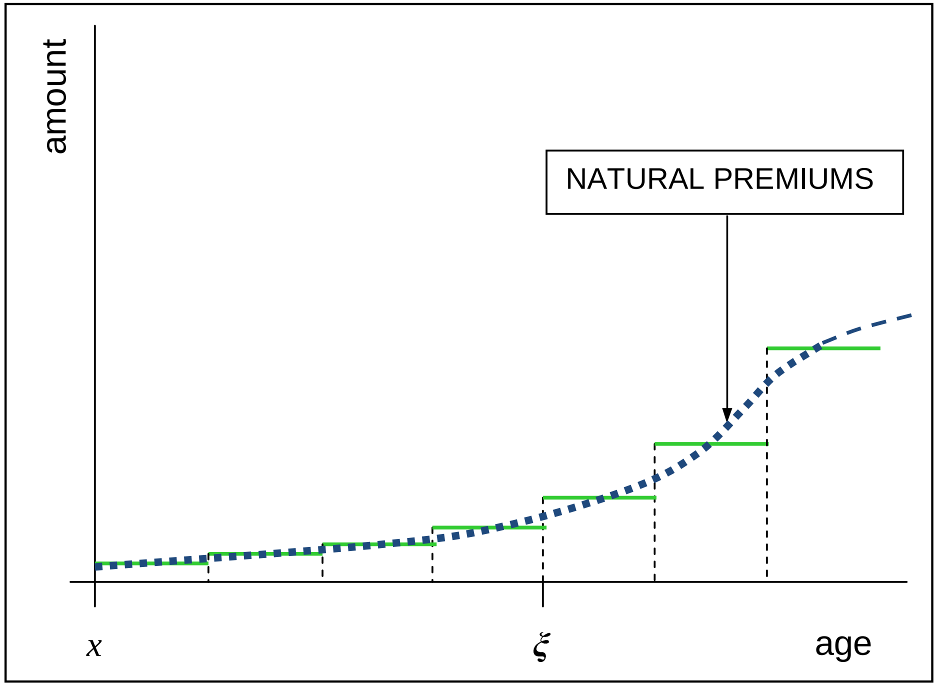
Risk transfer via insurance

Sequence of one-year covers, or multi-year cover with natural premiums



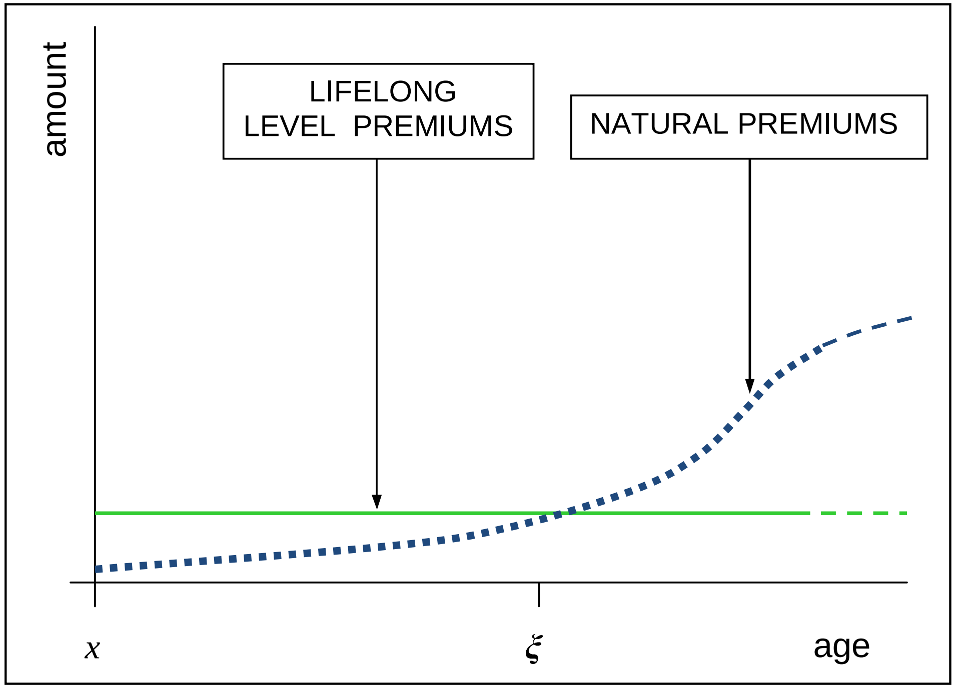
Health-related expected costs and natural premiums (including safety loading)

Sequence of temporary insurance covers, each cover financed via level premiums

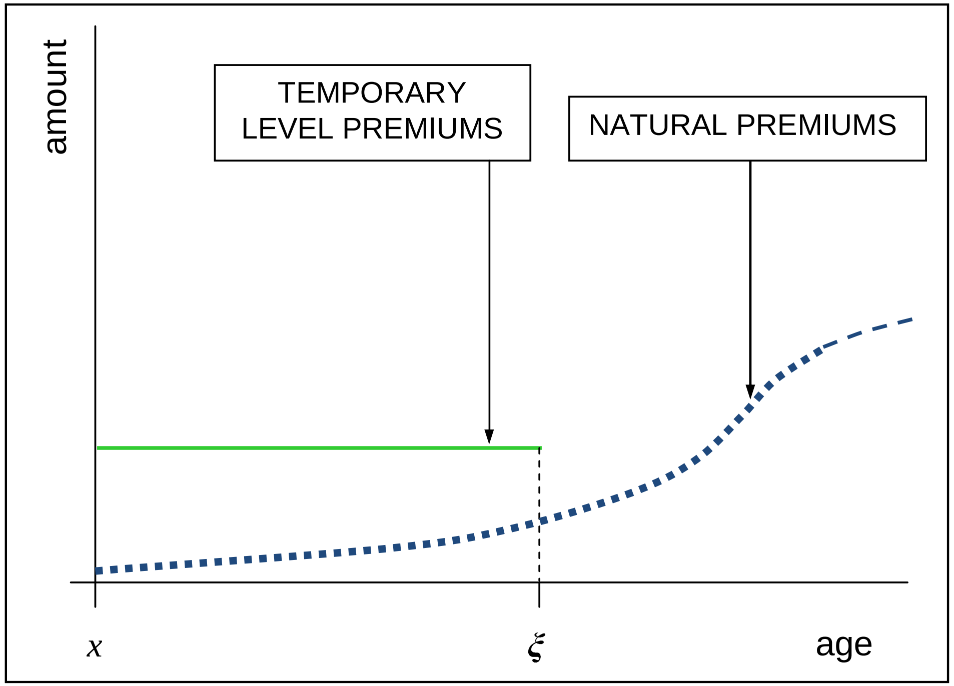


Temporary cover: natural premiums and level premiums)

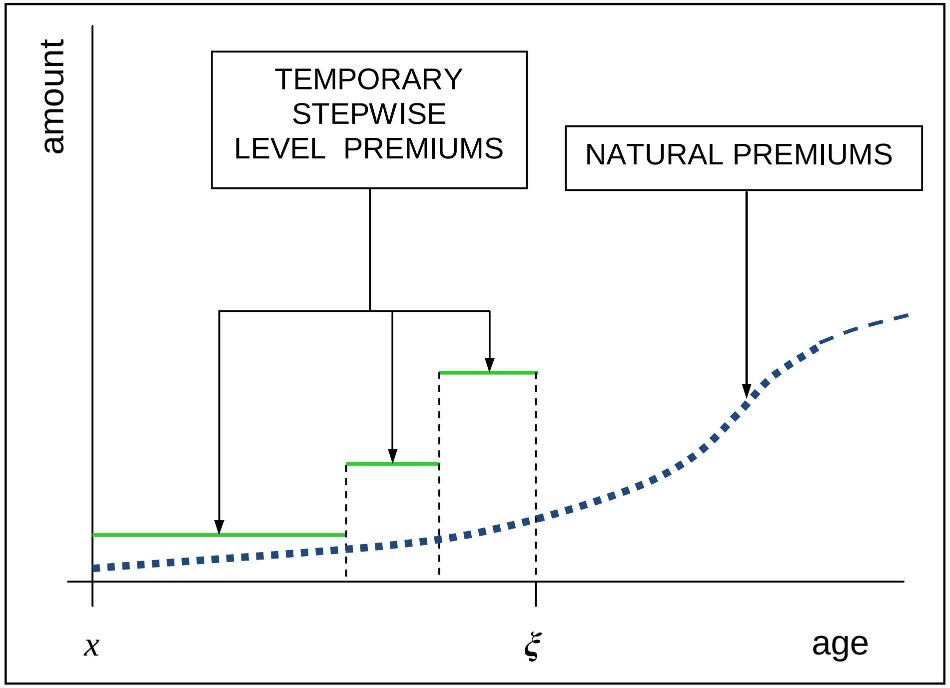
Lifelong cover



Lifelong cover: natural premiums and lifelong level premiums



Lifelong cover: natural premiums and temporary level premiums



Lifelong cover: natural premiums and temporary stepwise level premiums

FINANCING HEALTH-RELATED EXPENSES

Purposes of health insurance

- Replace random costs with sure costs (insurance premiums)
  - ▷ covering the risk via pooling
    also in one-year covers, or multi-year covers with natural premiums
- Limit consequences of time mismatching between income profile and health-related cost profile
  - ▷ pre-funding and risk coverage
    in long-term covers (possibly lifelong), with "premium levelling" (level, stepwise level, etc.)

Available alternatives for individual health costs financing

			Pre-funding	Pooling
1 Out-of-pocket			No	No
2 Savings			Yes	No
3 Insurance	3.1 one-year		No	Yes
	3.2 multi-year	3.2.1 natural premiums	No	Yes
		3.2.2 level premiums	Yes	Yes

Effects of various alternatives for health costs financing

Typical strategies for health costs financing

Health-related event		Appropriate financing strategy
Probability	Cost	
High	Low	Out-of-pocket
Medium	Medium	Savings
Low	High	Insurance

Choosing the strategy according to the "probability / cost" logic
(or "frequency / severity" logic)

HEALTH INSURANCE PRODUCTS

INTRODUCTION

Health insurance: a large set of insurance products providing benefits in the case of need arising from

- *accident*
- *illness*

and leading to

- ▷ *loss of income* (partial or total, permanent or non-permanent)
- ▷ *expenses* (hospitalization, medical and surgery expenses, nursery, etc.)

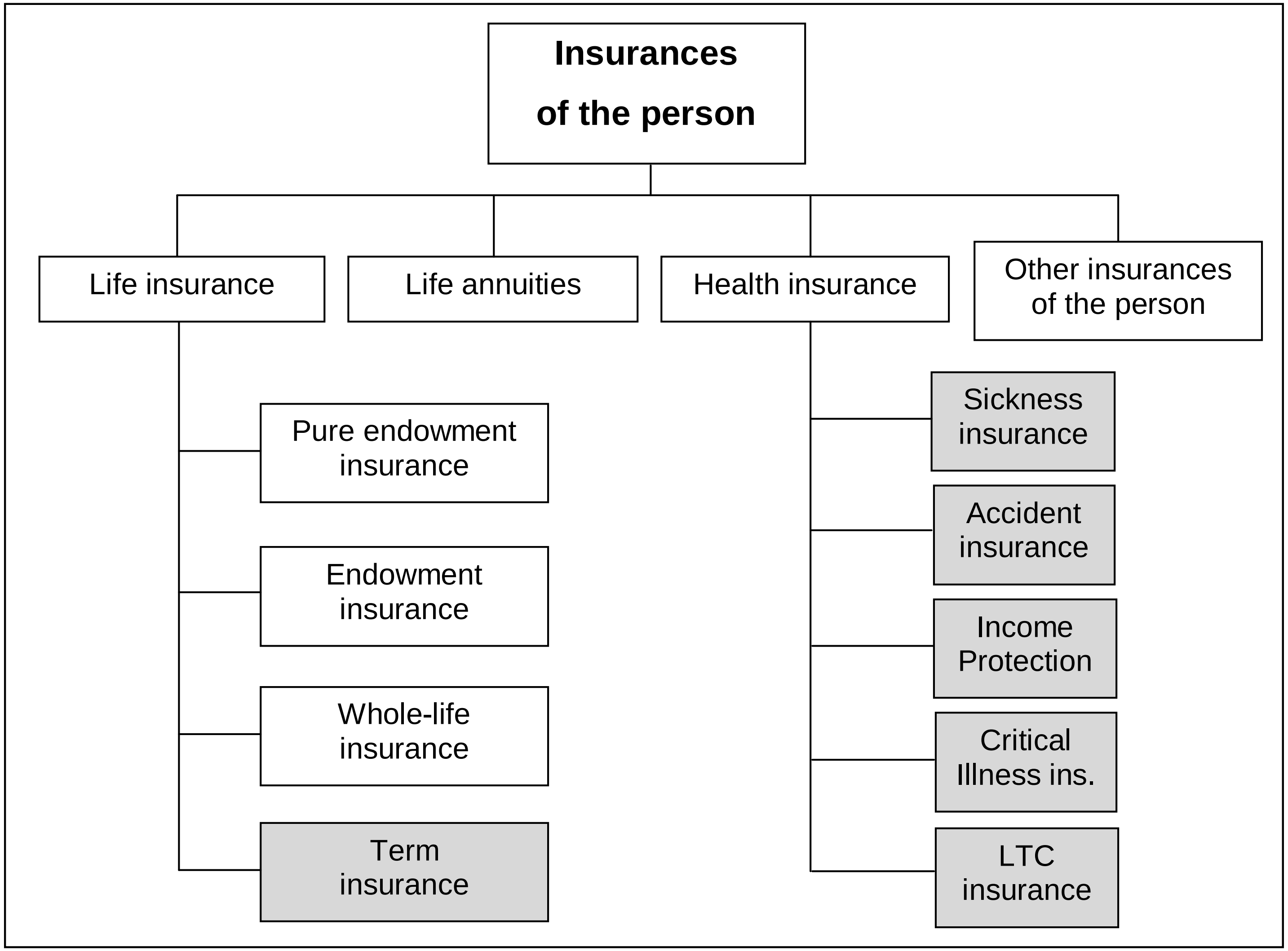
Health insurance belongs to the area of *insurances of the person*, which includes

- *life insurance* (in a strict sense): benefits are due depending on death and survival only, i.e. on the insured's lifetime
- *health insurance*: benefits are due depending on the health status, and relevant economic consequences (and depending on the lifetime as well)
- *other* insurances of the person: benefits are due depending on events such as marriage, birth of a child, education and professional training of children, etc.

See following Figure (shaded boxes $\Rightarrow$ *protection*)

Health insurance products are usually shared by "life" and "non-life" lines, according to local legislation and regulation

Health Insurance products *(cont'd)*



Insurances of the person: basic products

PRODUCTS AND TYPES OF BENEFITS

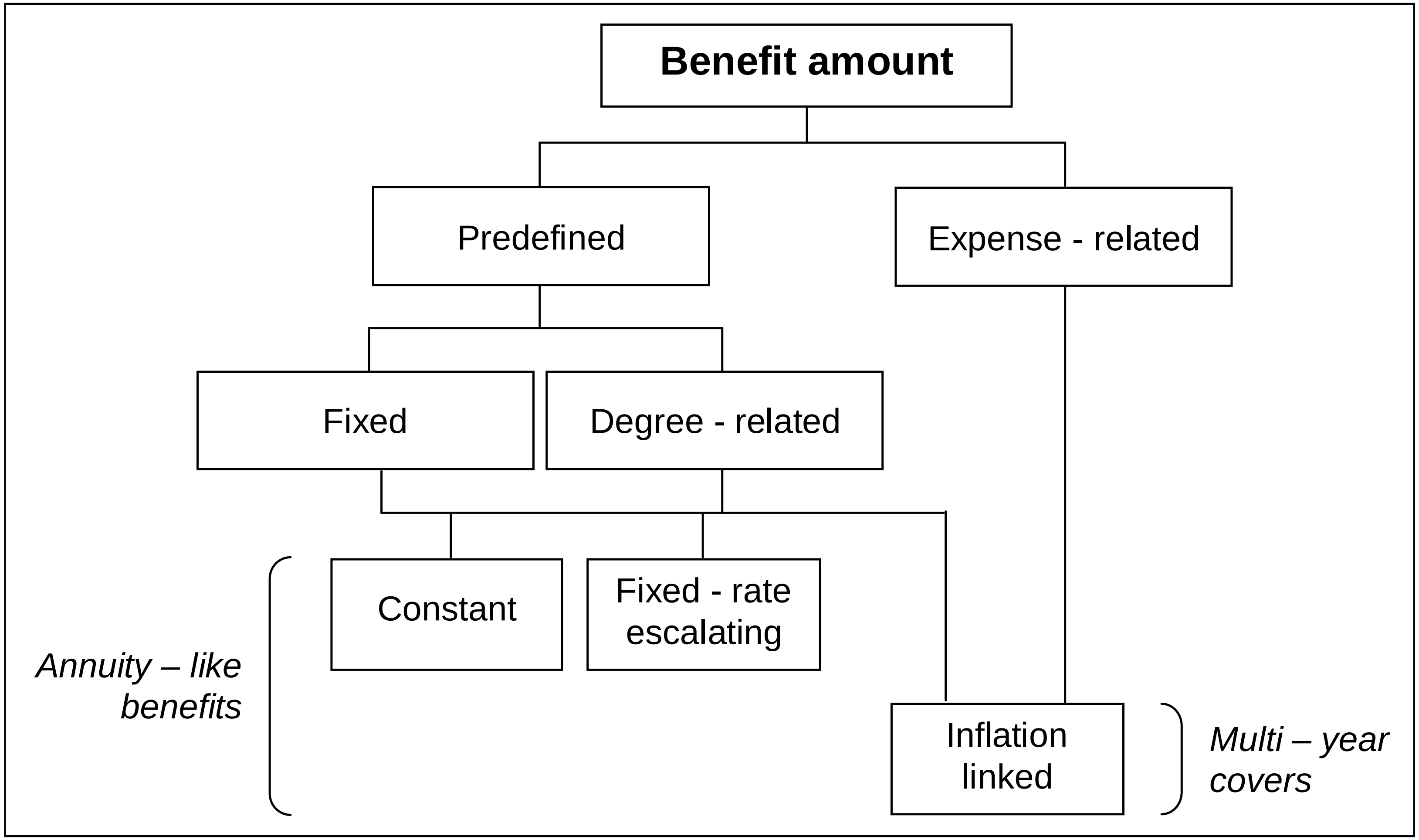
Monetary benefits and service benefits

Monetary benefits

- *Reimbursement benefits* designed to meet (totally or partially) health costs, for example medical expenses $\Rightarrow$ *expense-related benefits*; limitations: deductibles, limit values, etc.
- *Predefined benefit*: amount stated at policy issue
  - ◊ lump sum benefits
  - ◊ annuity benefits (for example to provide an income when the insured is prevented by sickness or injury from working)

  - ▷ *fixed-amount benefits*: independent of the severity of the health-related event and possible consequent costs
  - ▷ *degree-related benefits* (or *graded benefits*): amount linked to the severity of the health status expressed by some degree, e.g. the degree of disability

Health Insurance products *(cont'd)*



Defining the benefit amount: a classification

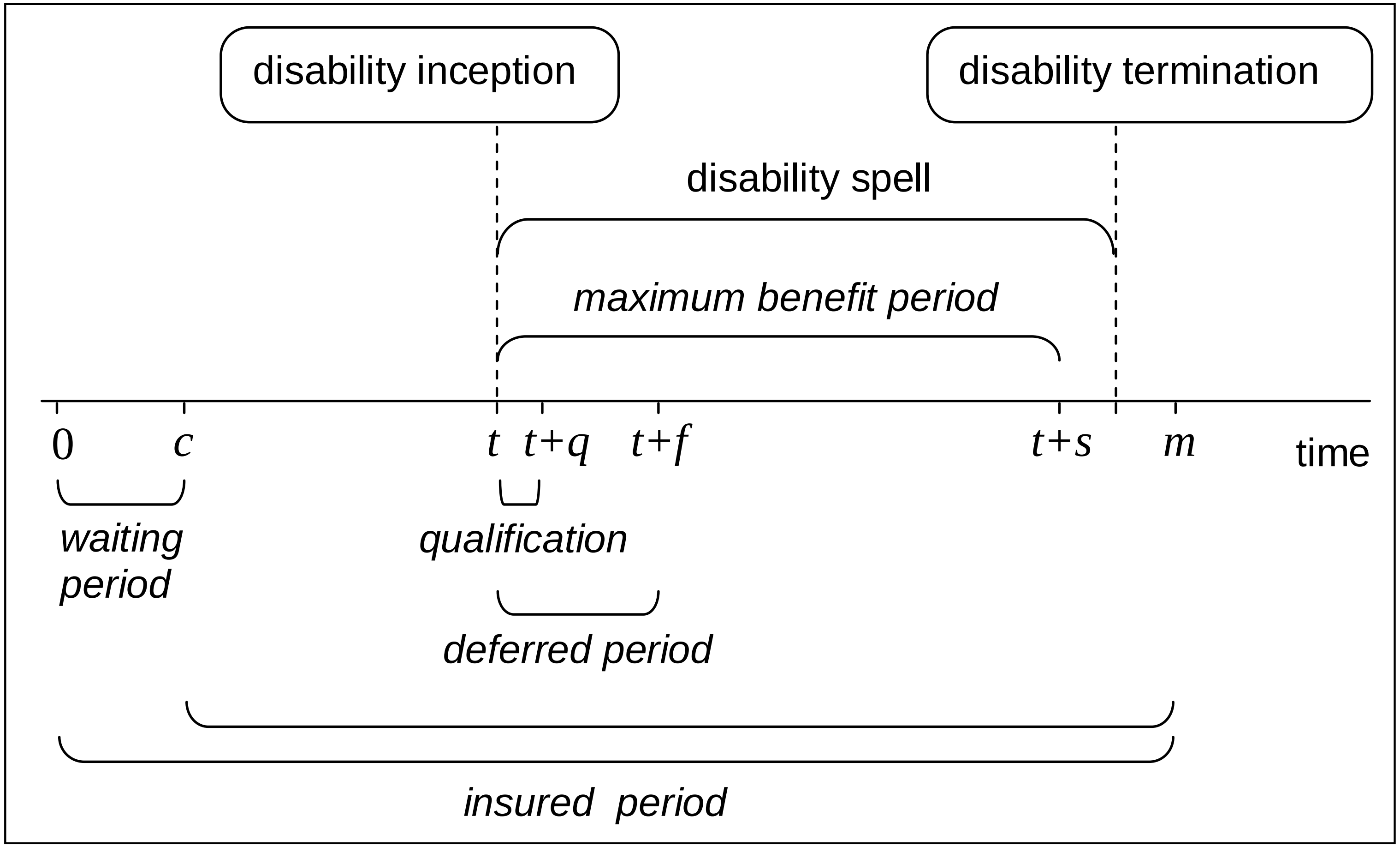
Service benefits

Care service provided by the insurer, relying on agreement between care providers (e.g. hospitals) and the insurer

Special type of long-term care service benefit is provided in the US by the CCRC's (Continuing Care Retirement Communities)

Policy conditions

- *Policy term*
  - ▷ one-year (or even shorter)
  - ▷ multi-year (possibly lifelong)
- *Exclusions*: limited set of causes leading to benefit payment (e.g. expenses not related to hospitalization can be excluded)
- *Limitations on the benefit amount*
  - ▷ limit value (maximum amount)
  - ▷ franchise
  - ▷ deductible (either amount or percentage)
- *Limitations on the benefit spell*
  - ▷ applied to annuity-like benefits
  - ▷ waiting period, deferred period, etc. (see following Figure)



Some policy conditions

PERSONAL ACCIDENT INSURANCE

Accident: "unintended, unforeseen, and/or violent event, which directly causes bodily injuries"

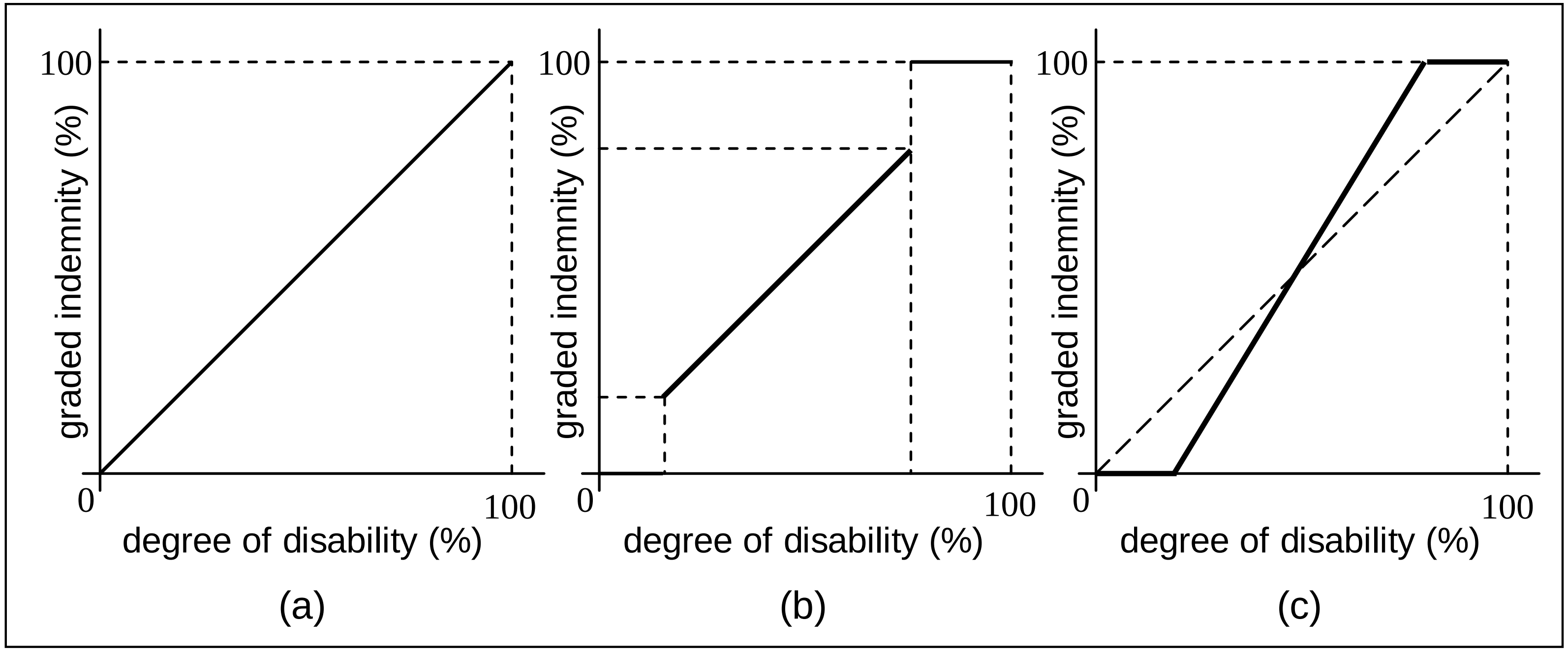
Type of benefits

Benefits provided by accident insurance policies

- *Death benefit* = lump sum paid in the case the insured dies as a result of an accident
- *Permanent disability benefit* = lump sum paid in the case of dismemberment
  - ▷ degree-related benefit: lump sum determined according to a benefit schedule

    Examples of degree-related benefits: see following Figures

    (a) no deductible
    (b) franchise deductible
    (c) deductible with "adjustment"



Degree-related benefit in Personal accident insurance

- *Reimbursement of medical expenses* (related to a covered accident)
- *Daily benefit*
  - ▷ fixed-amount benefit paid during the disability spells, caused by accident
  - ▷ maximum payment duration (e.g. 150 days, 300 days, 1 year)

Other features

Usually one-year covers (but in the case of riders to life insurance policies)

Qualification period applied for permanent disability benefit

Exclusions (war-related accidents, etc.)

Special insurance plans (professional accidents, travel accidents, etc.)

SICKNESS INSURANCE

Benefits paid in the event the insured becomes sick

Extent of benefits and level of coverage vary depending on policy conditions

Types of benefits

- *Reimbursement of medical expenses*

  Benefit package can include various expense-related items; in particular:

  - ▷ hospital inpatient $\Rightarrow$ all services provided while the insured is hospitalized, including surgery, lab tests, drugs, etc.
  - ▷ outpatient $\Rightarrow$ services provided in physician's office and hospital outpatient setting, including minor surgery
  - ▷ lab tests, drugs, physician prescribed, . . .

  Various policy conditions usually applied: waiting period, deductible, etc.

- *Temporary disability benefit* = daily benefit, in the case of disability caused by sickness
- *Permanent disability benefit* = lump sum
- *Hospitalization benefit*
  - ▷ daily benefit paid during hospital stays
  - ▷ non expense-related

Other features

Underwriting requirements (higher premiums for substandard risks)

Waiting period (to avoid possible adverse selection)

Qualification period for permanent disability benefit

Some policy conditions in medical expense reimbursement policy

(1) *Deductible* (also called *flat deductible*, or *fixed-amount deductible*): a predefined amount that the insured has to pay out-of-pocket before the insurer will (partially) cover the remaining eligible expenses; can either refer to each single claim (sickness or injury), or to the policy period (e.g. the policy year)

(2) *Proportional deductible* (also called *fixed-percentage deductible*, or *coinsurance*): fraction of eligible medical expenses that the insured has to pay, after having met the flat deductible

(3) *Stop-loss*: maximum amount the insured will pay out-of-pocket for medical expenses; can be referred either to each single claim or to the policy period

(1) + (2) + (3) $\Rightarrow$ sharing of costs between insured and insurer

In what follows: refer to a claim

Notation:

- x = generic expense amount
- D = flat deductible
- α = proportional deductible
- SL = stop-loss amount
- M = amounts which depends on D, α, SL (see Eq. $(^*)$)
- u = out-of-pocket payment
- y = benefit paid by the insurer

Of course $u + y = x$

For $0 < \alpha \leq 1$:

$$u = \begin{cases} x & \text{if } x < D \\ \alpha\,(x - D) + D & \text{if } D \leq x < M \\ SL & \text{if } x \geq M \end{cases}$$

$$y = \begin{cases} 0 & \text{if } x < D \\ (1-\alpha)\,(x-D) & \text{if } D \le x < M \\ x - SL & \text{if } x \ge M \end{cases}$$

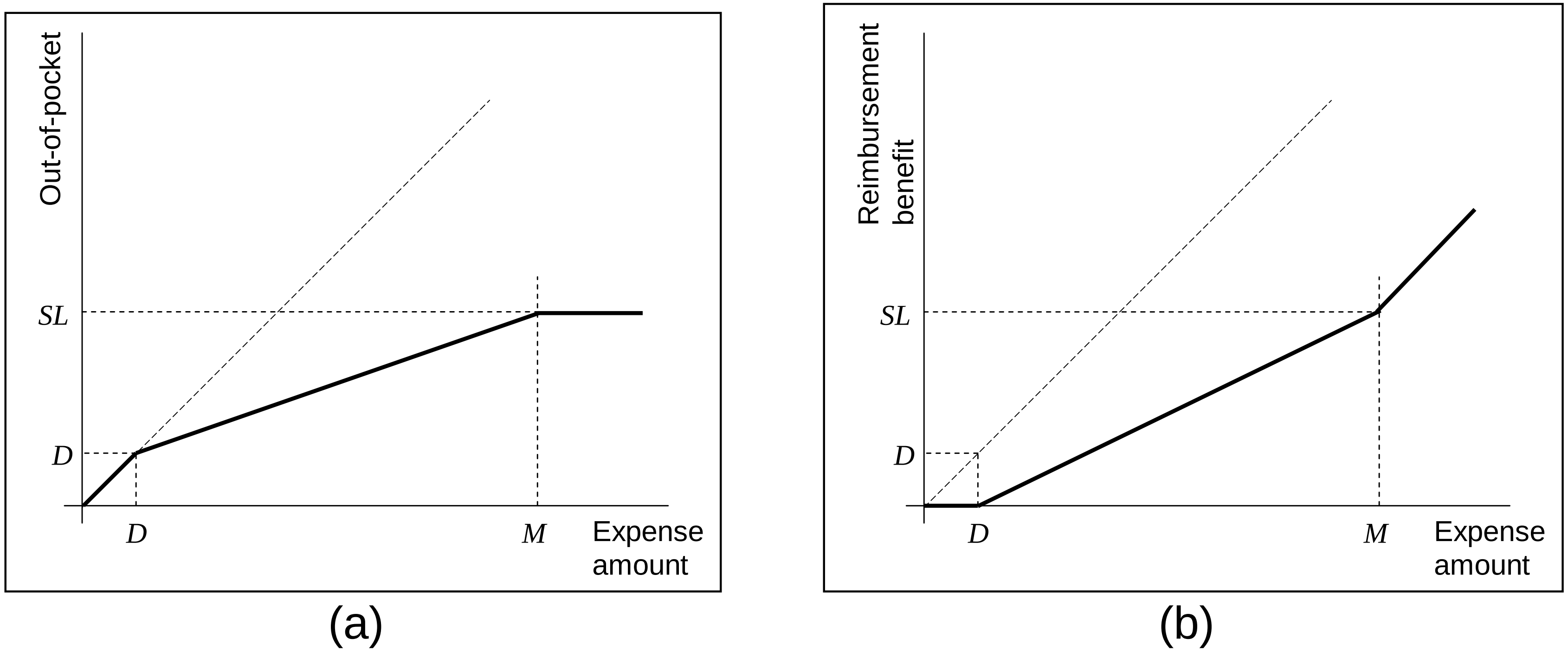
where

$$M = \frac{1}{\alpha}\big(SL - (1-\alpha)\,D\big) \qquad (*)$$

In particular:

$$M = SL \text{ if } \alpha = 1$$

$$M = \frac{1}{\alpha}\,SL \text{ if } D = 0 \text{ and } 0 < \alpha \le 1$$



Sharing of the cost

Example

Assume: $D = 100$, $\alpha = 0.25$, $SL = 500$

We find : $M = 1\,700$

Expense amount x	Out-of-pocket u	Reimbursement benefit y
50	$50 \ (= 100\%)$	0
300	$150 \ (= 50\%)$	150
900	$300 \ (= 33\%)$	600
$1\,800$	$500 \ (= 28\%)$	$1\,300$

Sharing medical expenses: examples

DISABILITY INSURANCE & INCOME PROTECTION (IP)

Several types of coverage in case of temporary and/or permanent disability

Types of benefits

- *Periodic income* (usually weekly or monthly) to an individual if he/she is prevented by sickness or injury from working
  $\Rightarrow$ *Income Protection, IP* $\Rightarrow$ annuity-like benefit
- *Lump sum* in the case of permanent disability
- *Waiver of premium*: rider benefit in a basic life insurance policy
  $\Rightarrow$ premiums waived during disability spells

We focus on IP

Various possible definition of disability; in particular:

(a) the insured is unable to engage in his/her own occupation

(b) the insured is unable to engage in his/her own occupation or carry out another activity consistent with his/her training and experience

(c) the insured is unable to engage in any gainful occupation

Benefit amount and policy conditions in IP

Annual amount stated in policy conditions, with a (reasonable) constraint:

$$\frac{\text{annual IP benefit (+ other possible disability benefits)}}{\text{annual income when active}} \leq \alpha$$

for example, with $\alpha = 70\%$ (to limit moral hazard)

Several policy conditions, in particular regarding the insured period (or cover period) and the benefit payment duration

- Usually, presence of a deferred period (e.g. 3 or 6 months, or 1 year)
- Possible "integration" with a short-term disability cover (i.e. with a short maximum benefit period)
- Long-term insurance cover (e.g. up to retirement)
- Waiver of premiums during disability spells

Further conditions

- Decreasing annuity benefit (to encourage a return to gainful work)
- Amount of the benefit scaled according to the degree of disability, if partial disability is allowed

Example

UK Income Protection policy (amounts in GBP)

$$b' = \begin{cases} 0.60\,w & \text{if } w \leq 25\,000 \\ 15\,000 + 0.50\,(w - 25\,000) & \text{if } w > 25\,000 \end{cases}$$

$$b'' = \max\{180\,000 - b^{[\text{other}]}, 0\}$$

$$b = \min\{b', b''\}$$

with $b^{[\text{other}]}$ = disability benefits provided by other institutions

LONG-TERM CARE INSURANCE (LTCI)

LTCI insurance provides the insured with financial support, while he/she needs nursing and/or medical care because of chronic (or long-lasting) conditions or ailments ($\Rightarrow$ implying dependence)

Remark

Interest in analyzing LTCI products

- ▷ In many countries, elderly population rapidly growing because of increasing life expectancy and low fertility rates
- ▷ Household size is progressively reducing $\Rightarrow$ lack of assistance and care services provided to old family members of the family
- ▷ LTCI products are rather recent $\Rightarrow$ senescent disability data are scanty $\Rightarrow$ pricing difficulties
- ▷ High premiums (viz because of a significant safety loading) $\Rightarrow$ obstacle to the diffusion of these products
- ▷ Stand-alone LTCI product: only "protection" $\Rightarrow$ packaging of LTCI benefits with lifetime-related benefits can enhance propensity to LTCI

Measuring the severity of dependence

According to *ADL (Activities of Daily Living) method*, the following activities and functions are, for example, considered:

1. eating
2. bathing
3. dressing
4. moving around
5. going to the toilet
6. bowels and bladder

Simplest implementation: for each activity or function, individual ability is tested (0 / 1)

Total disability level (or *LTC score*) given by the number of activities or functions the insured is not able to perform

LTC score expressed in terms of *LTC state*. See following Table

LTC score; unable to perform:	LTC state	Graded benefit (% of the insured benefit)
3 activities	I	40
4 or 5 activities	II	70
6 activities	III	100

Benefit as a function of the LTC state

More complex implementations rely on the degrees of ability to perform the various activities (see OPCS in following Example)

IADL (Instrumental Activities of Daily Living) method, or *PADL (Performance Activities of Daily Living) method* $\Rightarrow$ individual ability to perform "relation" activities; for example: ability to use telephone, shopping, food preparation, housekeeping, etc.

Example

OPCS index: based on the degree of functional dependence in performing 13 activities (among which mobility, eating, drinking, etc.)

Index quantifying the overall disability of a generic individual calculated according to the following procedure:

1. degree p_j assessed for each activity j, $j = 1, 2, \ldots, 13$
2. let $p^{(1)}$, $p^{(2)}$, $p^{(3)}$ denote the three highest values among the p_j's ($p^{(1)} \geq p^{(2)} \geq p^{(3)}$)
3. overall degree, p, determined via a weighting formula:

$$p = p^{(1)} + 0.4\, p^{(2)} + 0.3\, p^{(3)}$$

4. value of $p \Rightarrow$ "category" and "level" of disability (also used in various statistical reports); see following Table

p	Category	Level
$0.5 - 2.95$	1	-
$3.0 - 4.95$	2	-
$5.0 - 6.95$	3	-
$7.0 - 8.95$	4	-
$9.0 - 10.95$	5	-
$11.0 - 12.95$	6	I
$13.0 - 14.95$	7	I
$15.0 - 16.95$	8	I
$17.0 - 18.95$	9	II
$19.0 - 21.40$	10	II

Disability categories and levels according to OPCS index

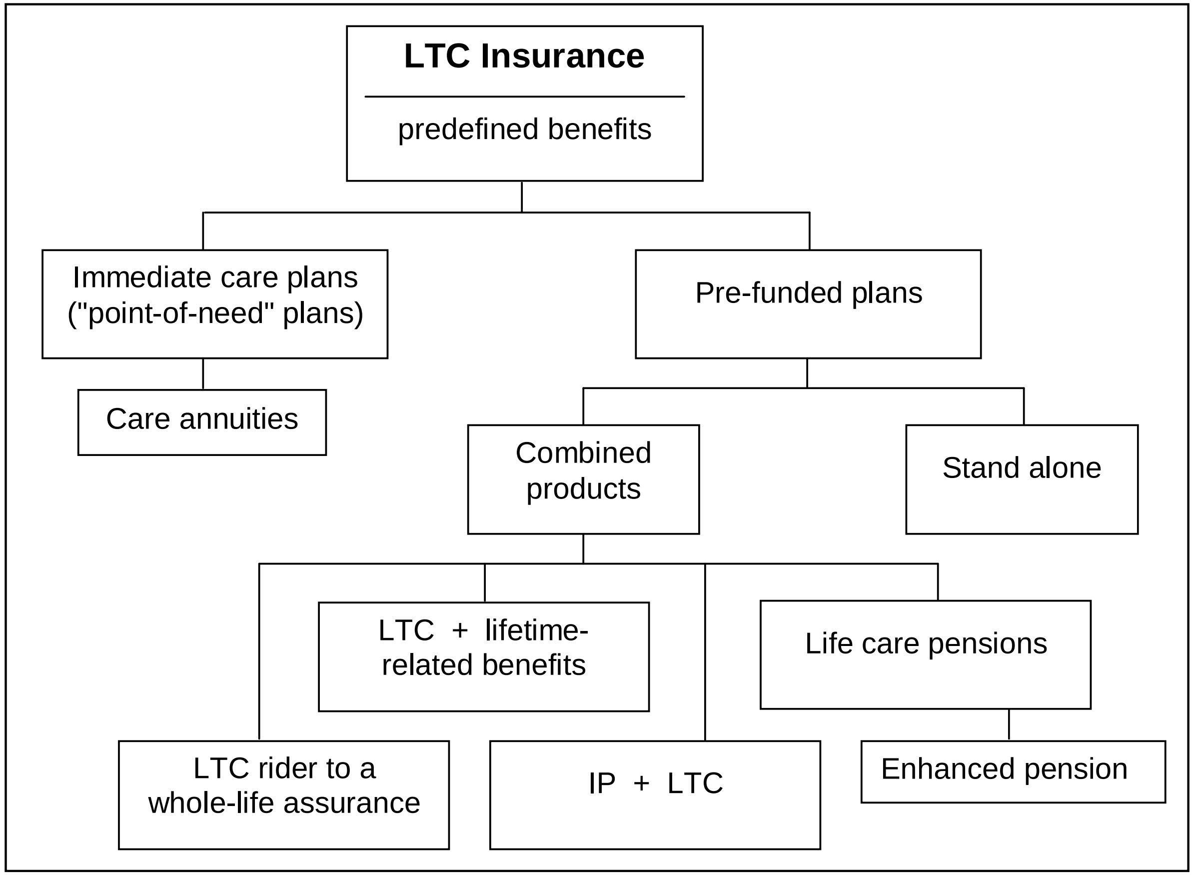
Remark

Critical aspect of disability assessment via ADL (or IADL): possible significant correlations among the individual ability to perform the various activities $\Rightarrow$ likely consequence: concentration of insureds in the "extreme" categories, i.e. those with either very low or very high disability degree

LTCI products: a classification

- Benefits with *predefined amount* (usually, a lifelong annuity benefit; in particular
  - ▷ *fixed-amount* benefit
  - ▷ *degree-related* (or *graded*) benefit, i.e. graded according to the the severity of the disability itself (for example, see Table)
- Reimbursement (usually partial) of nursery and medical expenses, i.e. *expense-related* benefits
- *Care service* benefits (for example provided by CCRCs)

Fixed-amount and degree-related benefits



A classification of LTCI products providing predefined benefits

Immediate care plans (or *care annuities*) relate to individuals already affected by disability (in "point of need")

Consist of:

- ▷ payment of a single premium
- ▷ an immediate life annuity (possibly degree-related)

Premium calculation based on assumptions of short life expectancy

Remark

Care annuities belong to the class of *special-rate annuities*, also called *underwritten annuities*, because of the ascertainment of higher mortality assumptions via underwriting requirements $\Rightarrow$ *substandard risk*
Special-rate annuities sold in several markets

- The underwriting of a *lifestyle annuity* takes into account smoking and drinking habits, marital status, occupation, height and weight, blood pressure and cholesterol levels
- *Enhanced annuity* pays out an income to a person with a slightly reduced life expectancy ("enhancement" comes from the use of a higher mortality assumption)
- *Impaired-life annuity* pays out a higher income than an enhanced annuity, as a result of medical conditions which significantly shorten the life expectancy of the annuitant (e.g. diabetes, chronic asthma, etc.)
- *Care annuities* are aimed at individuals, usually beyond age 75, with very serious impairments or individuals who are already in a LTC state

Pre-funded plans consist of:

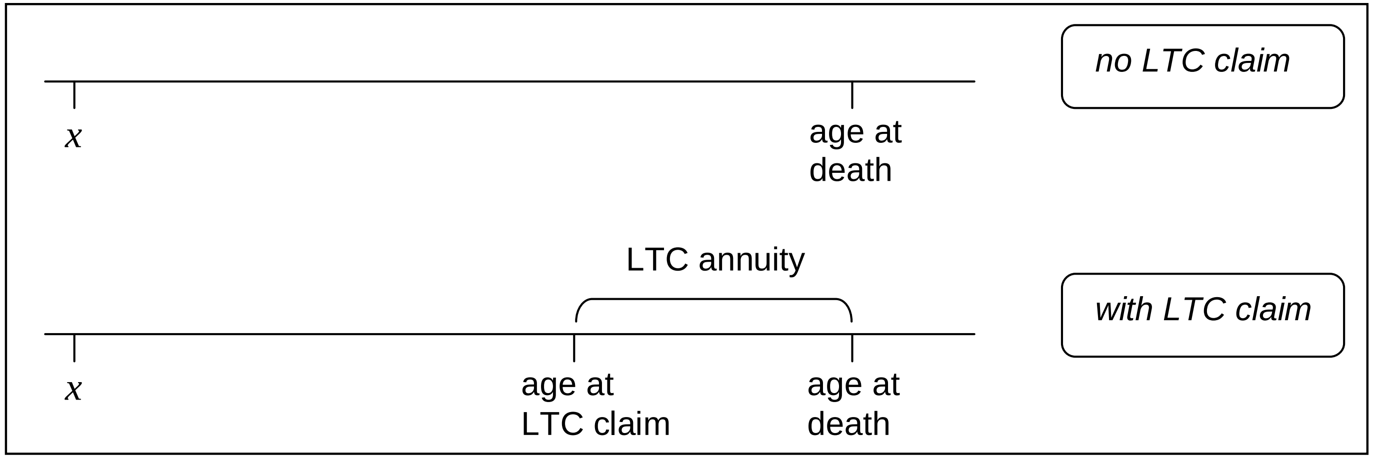
- ▷ accumulation phase, during which periodic premiums are paid (possibly a single premium)
- ▷ the payout period, during which LTC benefits (usually consisting in a life annuity) are paid in the case of LTC need.

Several products belong to the class of pre-funded plans

Stand-alone LTC cover

- ▷ benefit: annuity (possibly graded according to ADL)
- ▷ premiums: single premium, temporary annual premiums, lifelong annual premiums
- ▷ waiver of premiums on LTC claim
- ▷ insurance product providing a "risk cover" only

Two possible individual stories: see following Figure



LTC stand-alone annuity benefit: possible outcomes

Several examples of *combined products* $\Rightarrow$ LTC-related benefits combined with lifetime-related benefits

Aim: weakening the weight of the "risk component" by adding saving elements

Rider to a whole-life assurance policy

Annual benefit given by:

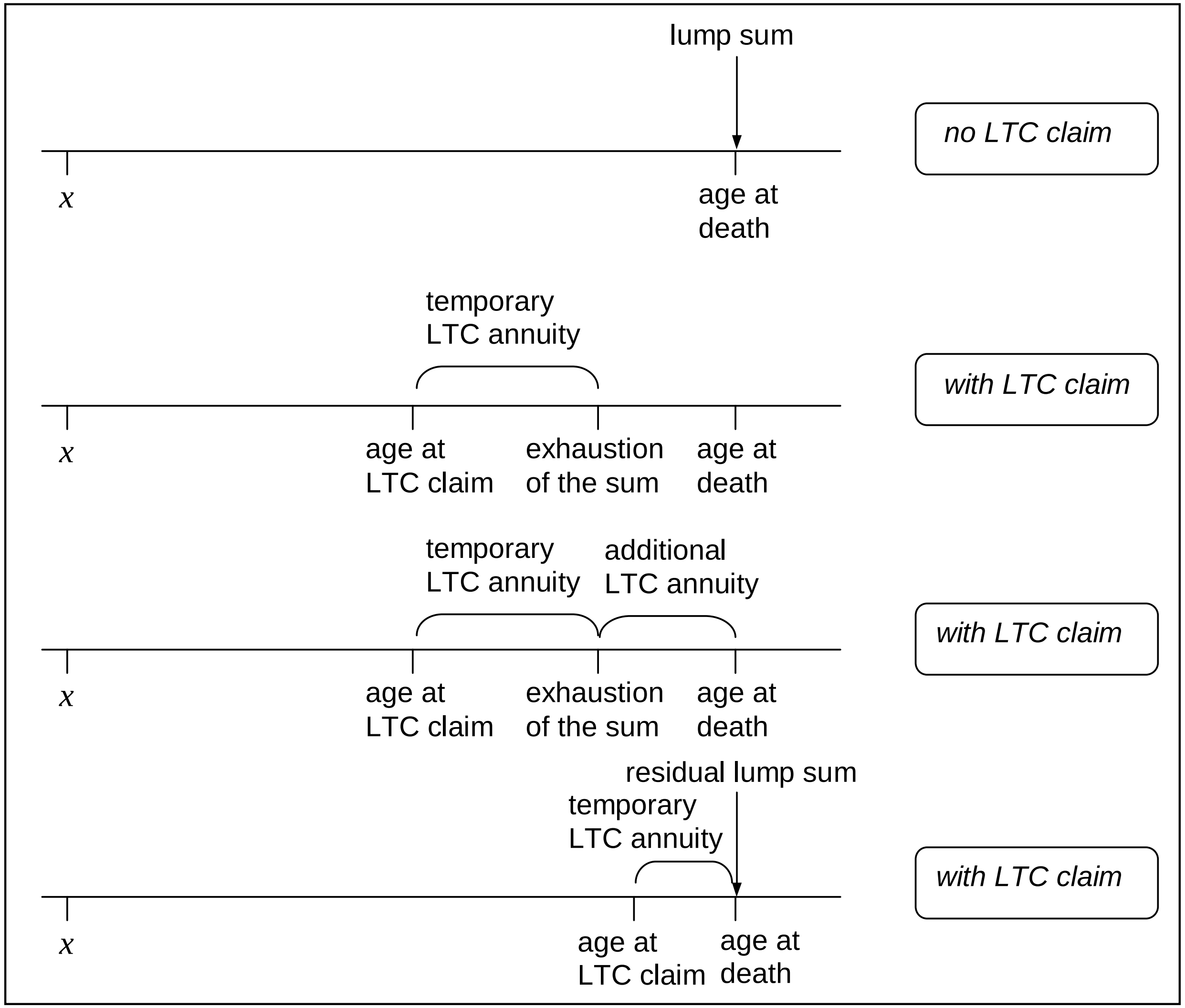
$$\text{annual benefit} = \frac{\text{sum assured}}{r}$$

paid for r years at most

- ▷ Death benefit consequently reduced, and disappears if all the r benefits are paid
- ▷ LTC benefit = *acceleration benefit*

LTC cover can be complemented by an additional deferred LTC annuity (financed by an appropriate premium increase) which will start immediately after possible exhaustion of the sum assured

Three possible individual stories: see following Figure



Whole-life assurance with LTCI as an acceleration benefit: possible outcomes

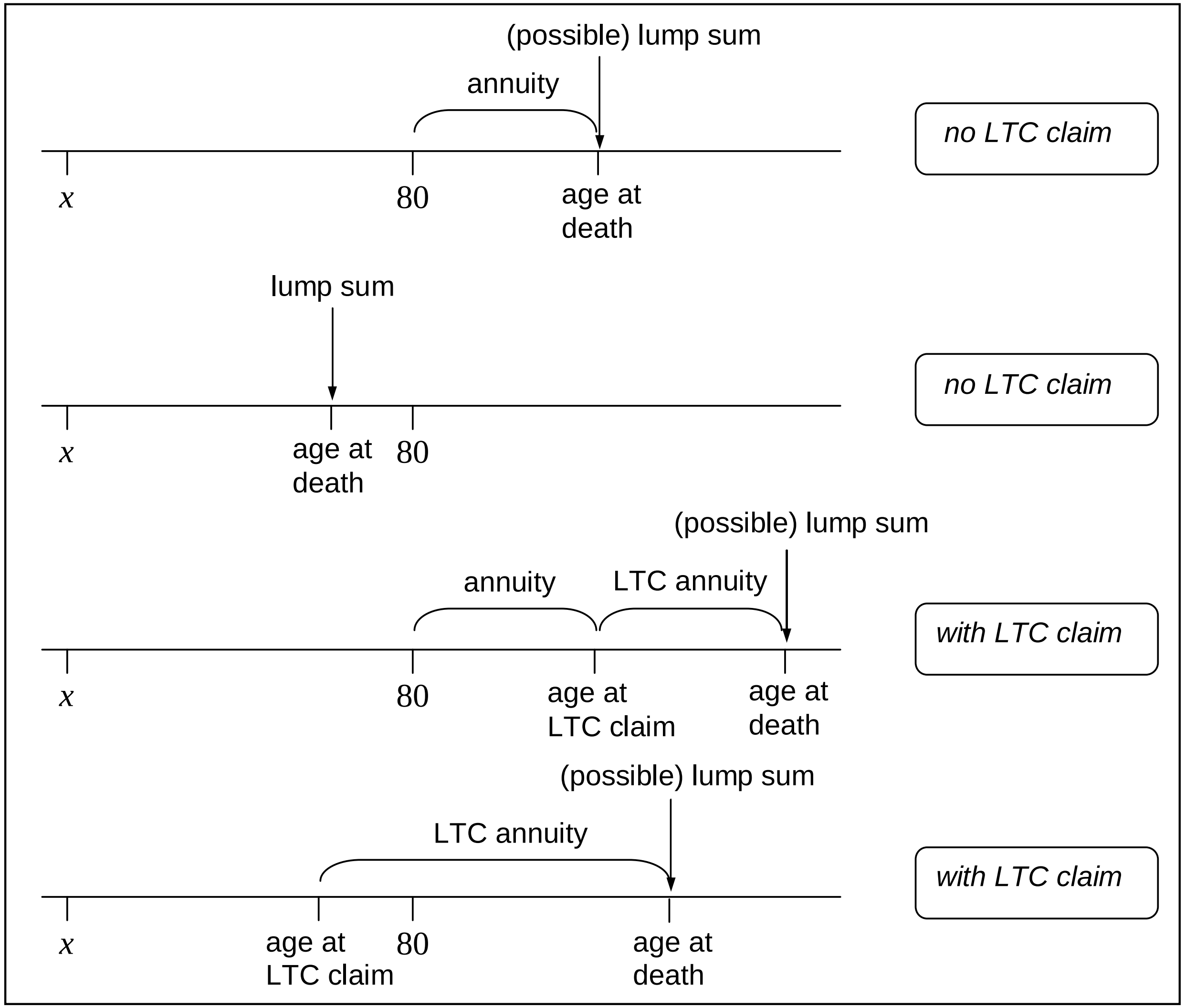
LTC cover combined with lifetime-related benefits

1. a lifelong LTC annuity (from the LTC claim on)
2. a deferred life annuity (e.g. from age 80), while the insured is not in LTC disability state
3. a lump sum benefit on death, alternatively given by
   (a) a fixed amount, stated in the policy
   (b) the difference (if positive) between a stated amount and the amount paid as benefit (1) and/or benefit (2)

Benefits 1 and 2 are mutually exclusive

Four possible individual stories: see following Figure

Health Insurance products *(cont'd)*



Insurance package including LTC annuity and lifetime benefits: possible outcomes

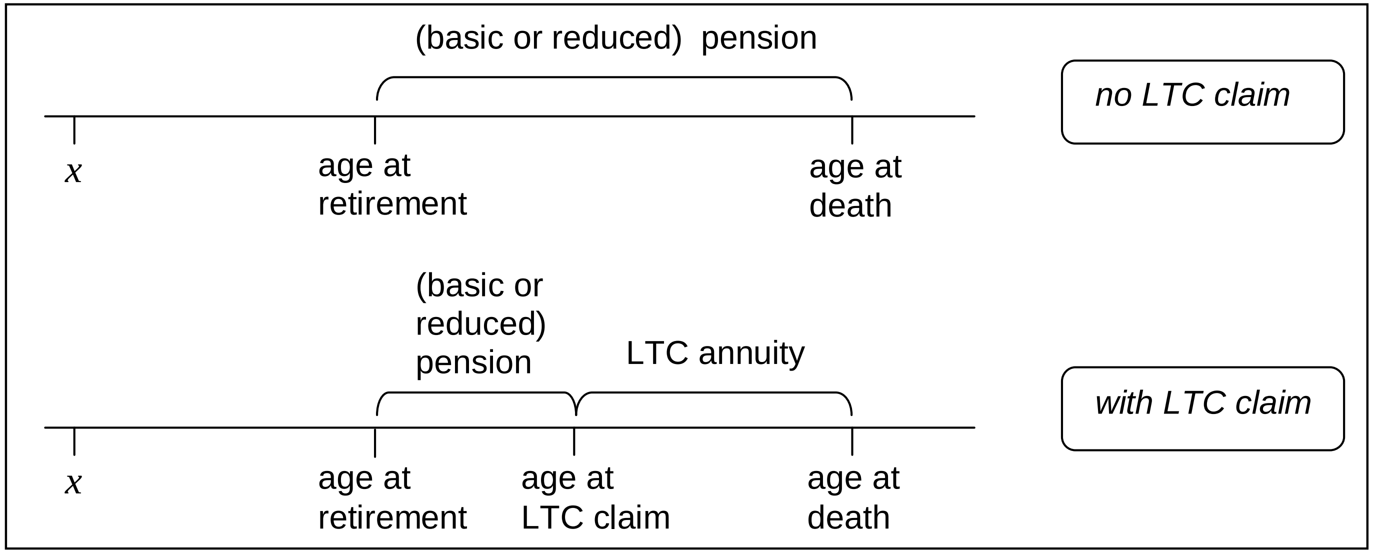
Life care pension (or *life care annuity*)

- ▷ LTC benefit defined as uplift with respect to the basic pension b
- ▷ basic pension b paid from retirement onwards, and replaced by the LTC annuity benefit $b^{[\mathrm{LTC}]}$ ($b^{[\mathrm{LTC}]} > b$) in case of LTC claim
- ▷ uplift financed during the whole accumulation period by premiums higher than those needed to purchase the basic pension b

Enhanced pension

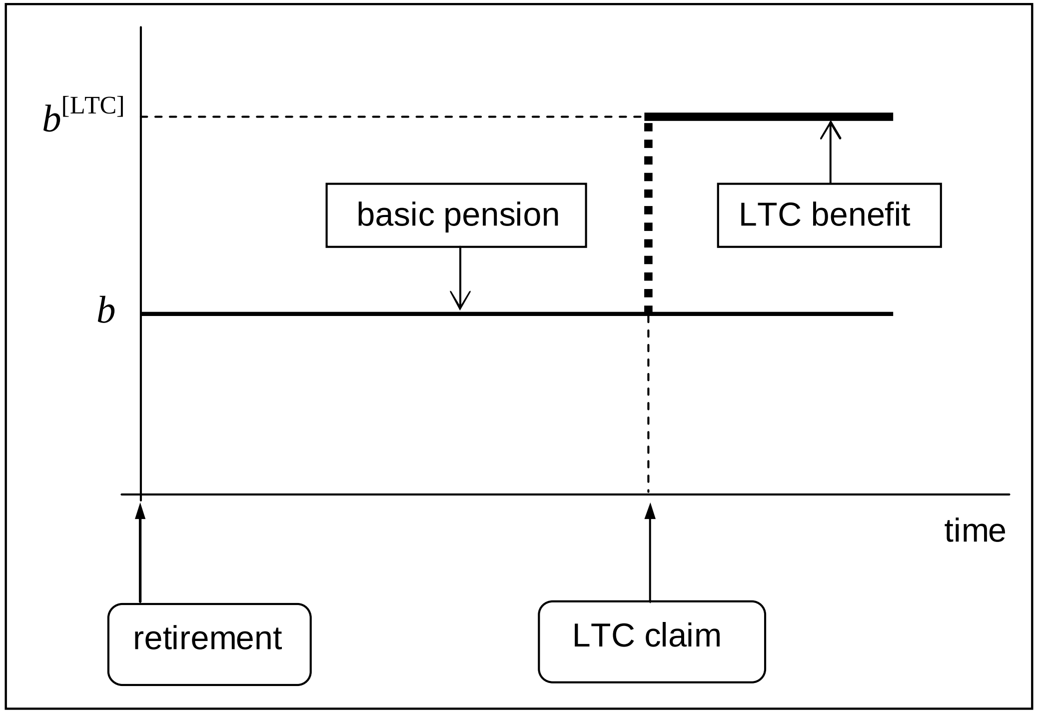
- ▷ a particular life care pension
- ▷ uplift financed by a reduction (with respect to the basic pension b) of benefit paid while the policyholder is healthy
- ▷ reduced benefit $b^{[\mathrm{healthy}]}$ paid as long as the retiree is healthy
- ▷ uplifted benefit $b^{[\mathrm{LTC}]}$ will be paid in the case of LTC claim ($b^{[\mathrm{healthy}]} < b < b^{[\mathrm{LTC}]}$)

See following Figure

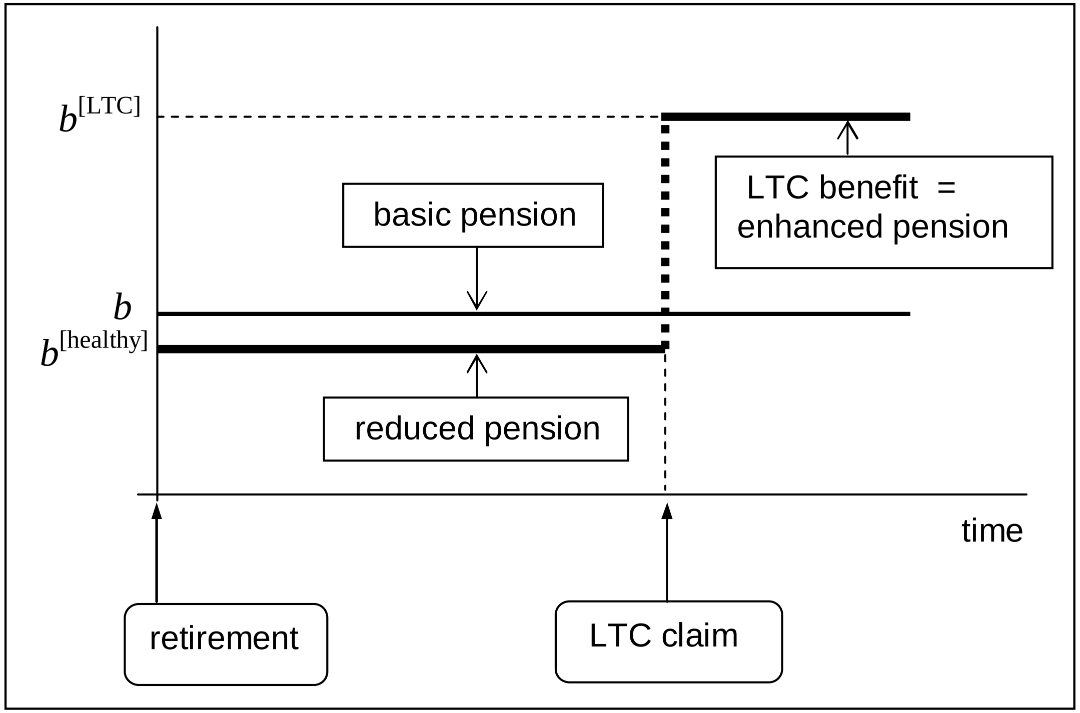


Life care pension and enhanced pension: possible otucomes

Benefits b, $b^{[\mathrm{healthy}]}$, $b^{[\mathrm{LTC}]}$: see following Figures



Benefits provided by a life care pension product



Benefits provided by an enhanced pension product

A life-long disability cover can include:

- Income Protection cover during working period (accumulation period for LTC benefits)
- LTC cover during retirement period

Expense-related benefits

Stand-alone LTC cover

- ▷ benefit: reimbursement of expenses related to LTC needs (nursery, medical expenses, physiotherapy, etc.)
- ▷ usually, limitations on eligible expenses
- ▷ usually, deductible and limit value

LTC cover as a rider to a sickness insurance

- ▷ resulting product: a whole-life sickness insurance
- ▷ extension of eligible expenses (e.g. nursing home expenses)
- ▷ daily fixed benefit for expenses without document evidence

Service benefits

LTCI products providing *care service benefits* usually rely on an agreement between an insurance company and an institution which acts as the care provider

Alternative: *Continuing Care Retirement Communities* (CCRCs), established in the US

- ▷ CCRCs offer housing and a range of other services, including long-term care
- ▷ Cost usually met by a combination of entrance charge plus periodic fees (that is, upfront premium plus monthly premiums)

CRITICAL ILLNESS INSURANCE (CII)

Very limited extension of the coverage, defined via listing (rather than via exclusions)

Diseases commonly covered: heart attack, coronary artery disease requiring surgery, cancer, and stroke

Type of benefits

Benefit: a fixed-amount lump sum, paid on diagnosis of a specified condition, rather than on disablement

- ▷ does not indemnify the insured against any specific loss due to medical expenses (medical expense reimbursement is provided by sickness insurance)
- ▷ does not meet any specific income need, arising from loss of earnings (which is met by an IP policy)

Benefit arrangements:

- *stand-alone cover*
  - ▷ only includes a CII benefit
  - ▷ the insurance policy ceases immediately after the payment of the sum assured
- *rider benefit to a basic life policy including death benefit*
  - ▷ *acceleration benefit*
    - ◊ a share of (or all) the sum insured in the basic life policy is paid on critical illness diagnosis
    - ◊ the (possible) remaining sum is payable on death, if this occurs within the policy term
  - ▷ *additional benefit*: the insurance policy includes two separate covers (possibly with different sum assured)
    - ◊ one paying the sum assured in the case of death
    - ◊ the other paying the sum assured in the case of critical illness

Multiple critical illness benefits

Need for protection against further possible serious illnesses can last beyond the (first) claim

Insurance products providing coverage extended to more than one critical illness claim can provide a more complete protection

Two alternative approaches

- Multiple CII benefits provided by a *buy-back CII product*
  - ▷ a classical CII product with a "buy-back" option as a rider
  - ▷ right to reinstate the CII cover after the first claim
  - ▷ (second) CII cover sold without medical assessment and without change in the premium rates, after a waiting period (1 year, say) following the first claim
  - ▷ the option must be chosen at policy issue
  - ▷ usually, the same or related type of illness is excluded from the second coverage

- Specific *multiple CII cover*, usually designed as a stand-alone cover
  - ▷ "grouping approach" usually adopted $\Rightarrow$ classify the diseases and determine appropriate exclusions
  - ▷ in general, after a claim due to a disease belonging to a given group, all the diseases included in that group (and hence highly correlated) are excluded from further coverage

OTHER LIMITED-COVERAGE PRODUCTS

CII is an example of “limited-coverage” insurance product. Other examples follow

Cancer insurance policy

Can be shaped in several different ways, also depending on the specific insurance market

Underwriting requirements are applied

Two main types of benefit:

- *lump sum benefit* consists of a single payment upon the diagnosis of a cancer $\Rightarrow$ fixed-amount benefit, which can be used in any way, not necessarily related to medical expenses the insured incurs (for example: ground and air transportation, private nursing, etc.)

- *expense benefit plan* consists of a set of payments, each payment related to a specific expense item (medical tests, hospital stay, surgery, radiation, chemotherapy, etc.); the amount of each payment is predefined in the policy $\Rightarrow$ fixed-amount benefits (although the total amount paid-out depends on the specific needs)

Surgery cash plan

Provides the insured with a cash benefit in case of medically necessary in-patient or day surgery

A waiting period is commonly applied to avoid adverse selection, whereas no particular underwriting requirements are usually applied (at least for given age ranges at policy issue) $\Rightarrow$ guaranteed-issue product

Basic benefit: a *lump sum benefit*, whose amount paid out depends on the sum insured, and then varies according to the severity of the operation and the recovery period required $\Rightarrow$ graded benefit

Note that:

- the cash benefit is not a reimbursement benefit
- the insured can use the cash amount for any purpose, including post-surgery care, physiotherapy, etc.

Possible supplementary benefit: a fixed-amount daily benefit, payable during the hospital stay

COMBINING HEALTH AND LIFE BENEFITS

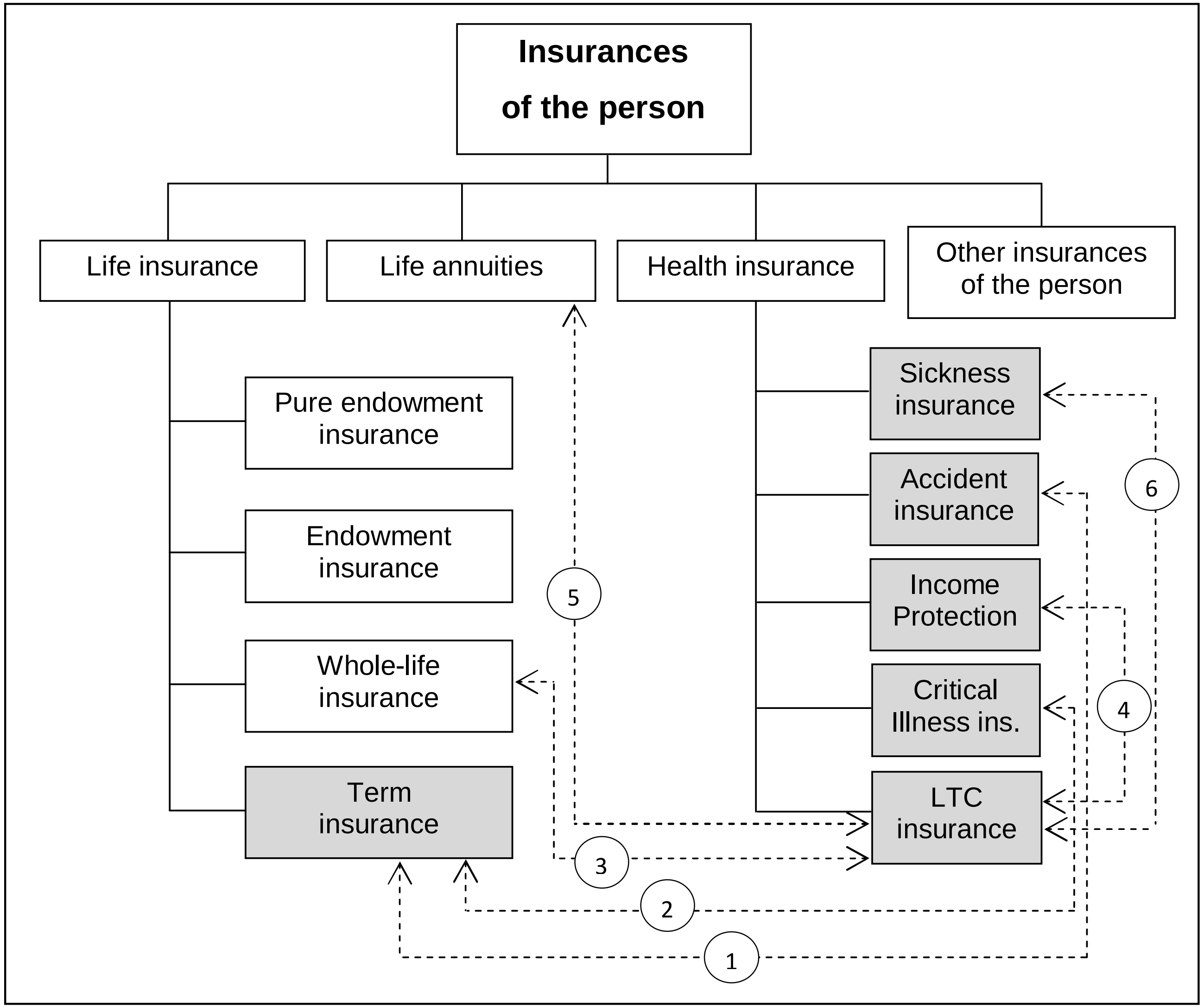
Insurer's perspective:

- a combined product can result profitable even if one of its components is not profitable
- packaging several insurance covers into one policy $\Rightarrow$ total amount of policy reserve can constitute a policy "cushion" for facing poor experience inherent one of the package components (provided that some degree of flexibility in using available resources is allowed)

Client's perspective:

- purchasing a combined product can be less expensive than purchasing each single component thanks to a reduction of
  - ▷ acquisition costs charged to clients
  - ▷ safety loading

Health Insurance products *(cont'd)*



Combining health and life benefits

Health covers as riders to life insurance

Examples

- Accident insurance benefits as riders to a life insurance policy which includes a death benefit (see link (1)); in particular:
  - ▷ sum insured as the death benefit paid in the event of permanent disability
  - ▷ in case of accidental death, amount higher than the sum insured as the (basic) death benefit
- Critical illness benefit as a rider to a term insurance (see link (2))
- Waiver of premiums as a rider benefit in several life insurance policies: premiums waived in the event of (total) disability, over the whole disability spell

Health covers in insurance packages

LTCI benefits in insurance packages

- ▷ with lifetime-related benefits (see link ③ and link ⑤)
- ▷ with other health-related benefits, for example with IP (link ④), or with lifelong sickness insurance (link ⑥)

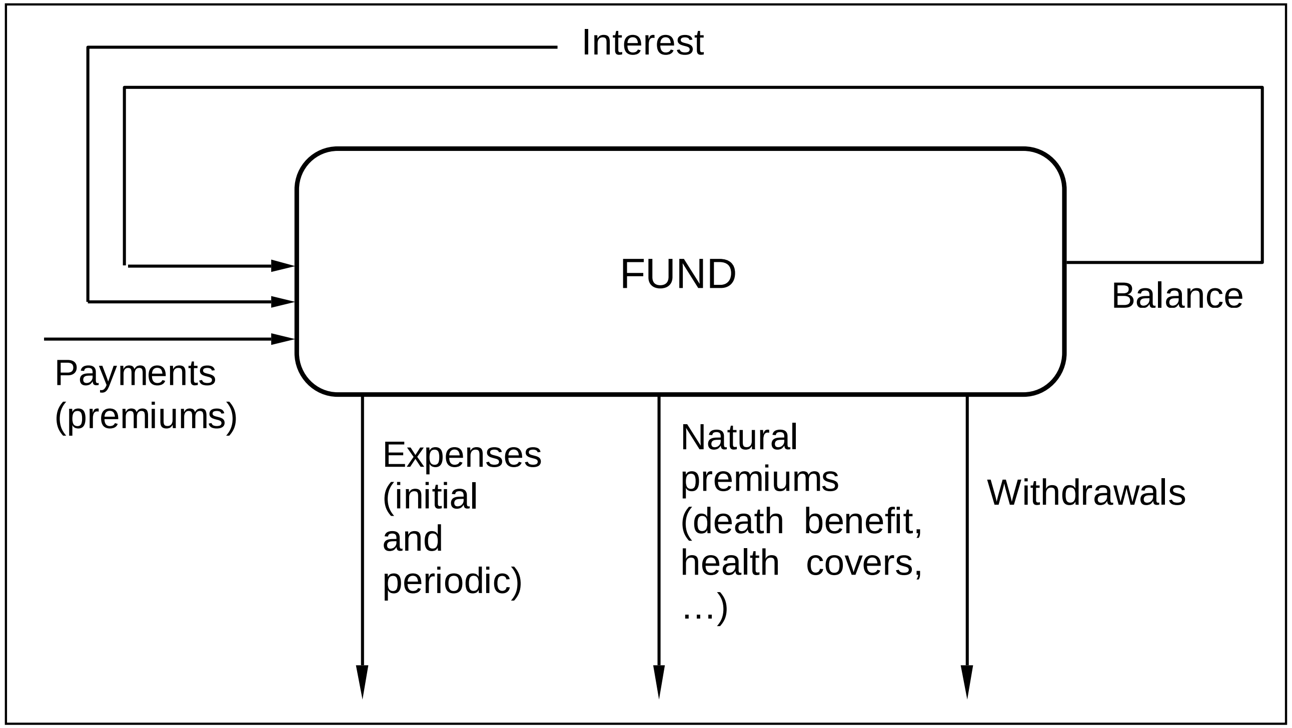
Universal Life (UL) policies are typical products of the US market

UL = insurance package in the context of the insurances of the person

Several health-related benefits can be included (lump sum in case of permanent disability, daily benefit in the case of temporary disability, medical expense reimbursement)

All benefits financed withdrawing the related periodic (e.g. annual) cost from the fund

See following Figure



Financing health insurance covers within a UL product

GROUP INSURANCE IN THE HEALTH AREA

Many health insurance products can be designed and sold on a group basis $\Rightarrow$ *health group plans*

Provide coverage to a select group of people (typically consisting of employees of a firm, possibly extended to their dependents)

Usual benefit package first includes

- medical expense reimbursement (dependents may be included)
- income protection

Health group insurance may be

- ▷ compulsory $\Rightarrow$ all employees are members of the plan $\Rightarrow$ no adverse selection
- ▷ voluntary $\Rightarrow$ all eligible employees may decide to opt for the group cover $\Rightarrow$ underwriting requirements

Health group plans can be placed in the framework of *employee benefit plans*

Provide benefits other than the salary, among which insurance-related benefits:

- death benefits, paid to the employee's dependents in the event of death during the working period
- pensions, i.e. post-retirement benefits
- health insurance covers

Traditional health group plans: benefit package and related limitations (exclusions, deductibles, etc.) defined in the group insurance policy
$\Rightarrow$ premium calculation follows

Alternative structure, implemented in the US in particular:
can be found in the *Defined Contribution Health Plans* (DCHPs).

DCHP arrangement:

- ▷ the employer pays a defined amount (that is, a contribution) to each employee
- ▷ the employee can then purchase individual health policies on the insurance market, according to his/her needs and preferences

DCHP can be implemented in different ways

- structure described above: *"pure" DCHP*, or *"individual market model" of DCHP*
- alternative structure: *"decision support model" of DCHP*
  $\Rightarrow$ employer's defined contributions fund for each employee
  - ◇ a health-savings account
  - ◇ a health insurance cover (usually with high deductibles) within a health group policy

PUBLIC AND PRIVATE HEALTH INSURANCE

A large variety of health insurance arrangements can be found looking at different countries

In particular, mixed systems of health care funding are rather common, which rely on both

- *public health insurance*, mainly financed through income-related taxation or contributions
- *private health insurance*, basically relies on insurance products financed through premiums whose amount depends on the value of the benefit package

Various "interactions" between public and private health insurance can be observed in different countries, as a result of the local legislation

For instance:

- participation into public health insurance scheme can be
  - ▷ mandatory
    - either for the whole population
    - or for eligible groups only
  - ▷ voluntary for specific population groups
- private health insurance
  - ▷ voluntary in most countries
  - ▷ a basic health coverage is mandatory in some countries

Classification of the main functions of health insurance products

1. *Primary private health insurance*: health insurance that represents the only available access to basic coverage for individuals who do not have public health insurance; in particular:
   (a) *principal* private insurance represents the only available access to health coverage for individuals where a public insurance scheme does not apply
   (b) *substitute* private insurance replaces health coverage which would otherwise be available from a public insurance scheme
2. Private insurance can offer *duplicate covers*, i.e. coverage for health services already provided by public insurance; also offer access to different providers or levels of service; does not exempt individuals from contributing to public insurance
3. *Complementary covers* complement coverage of publicly insured services or services within principal/substitute insurance (which pays a proportion of qualifying care costs) by covering all or part of the residual costs not otherwise reimbursed
4. *Supplementary covers* provide coverage for additional health services not covered by the public insurance scheme; extension depends on the local public health legislation (may include luxury care, long-term care, dental care, rehabilitation, alternative medicine, etc.)

MICROINSURANCE IN THE HEALTH AREA

Basic concept: "excluded population", that is, a population without participation or with inadequate participation in social life, or without a place in the consumer society

Examples, in several countries:

- people active in the informal economy in urban settings
- most of households in rural areas
- employees in small workplaces
- self-employed
- migrant workers

Exposure to accident and illness risks may be particularly significant among those people

Basic problem: making health insurance widely accessible

How to provide health insurance is a government choice $\Rightarrow$ in most cases the choice has been to rely on the insurance market

Health microinsurance can provide coverage of:

- ▷ illness and possible consequent hospitalization
- ▷ injury and possible related disability
- ▷ financial consequences of early death (death risk can constitute object of life microinsurance)

Several parties involved in the implementation of a microinsurance programme

Different models can be recognized

Various models (in what follows, arrangements 1 to 3) rely on a *health microinsurance scheme* (HMIS), i.e. an institution which provides insurance covers to individuals

Range of tasks assumed by HMIS and hence degree of its involvement in the delivery and management of the insurance covers, as well as the role of other possible parties, vary according to the arrangement adopted

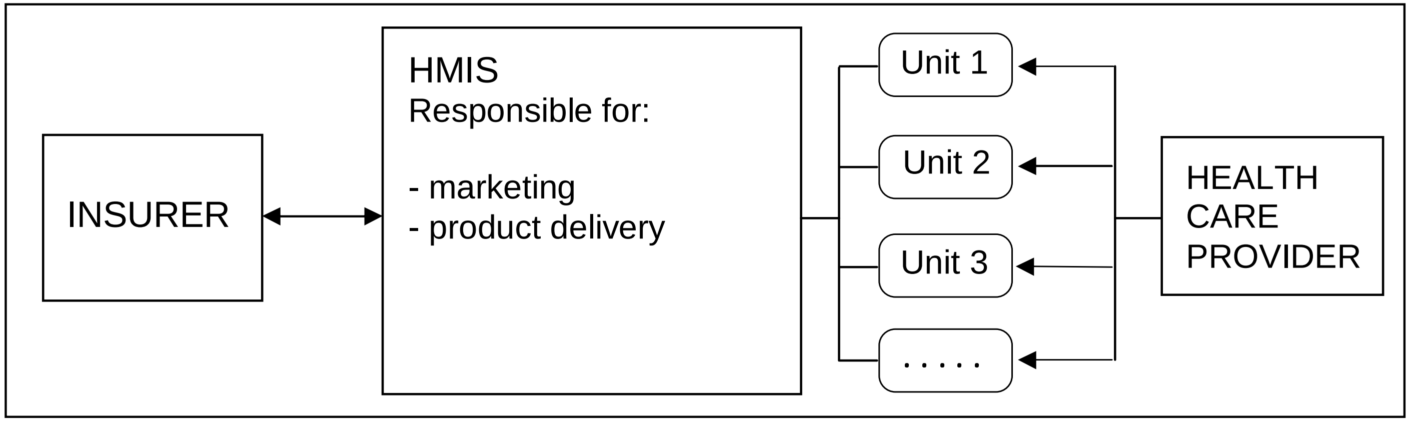
Ultimate target in most microinsurance arrangements: the *unit*, which consists of individuals sharing a common activity and/or living in a well defined geographic area

A microinsurance arrangement can involve several units
$\Rightarrow$ improvement in the diversification via pooling can be gained

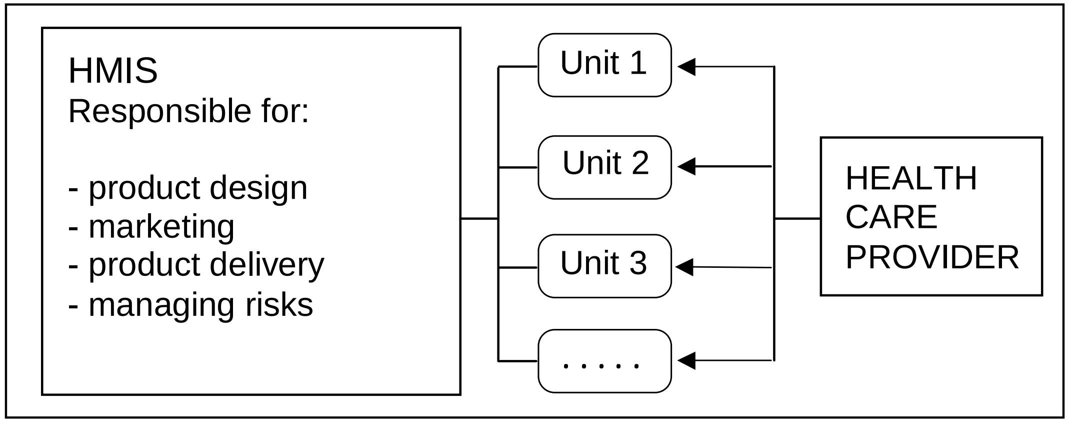
1. This arrangement relies on a partnership which involves, besides the HMIS, an insurance company and an institution acting as the health provider
   - HMIS responsible for:
     - ▷ marketing of the health insurance products (see below)
     - ▷ delivery of the products to the clients in the units
   - insurance company is responsible for
     - ▷ design of the insurance products (although the appropriate types of products should be suggested by the HMIS)
     - ▷ the management of risks transferred by the individuals belonging to the units
   - health care provider delivers services as hospitalization, surgery, etc.

   See Figure

Health insurance products *(cont'd)*



HMIS-based health microinsurance arrangement (1)



HMIS-based health microinsurance arrangement (2)

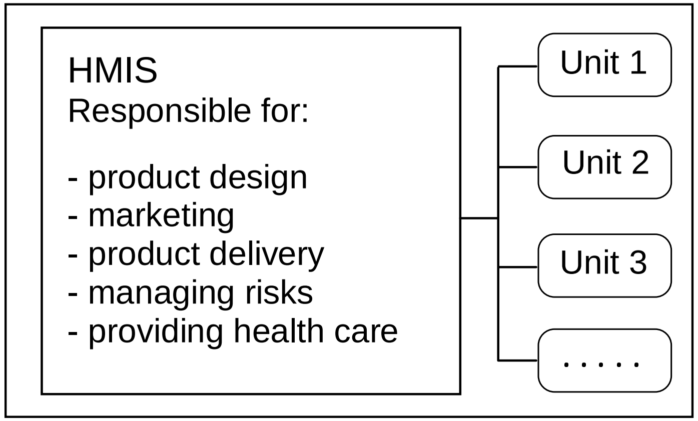
2. Partnership only involves (besides the HMIS) a health care provider
   - HMIS responsible for
     - design of the health insurance products
     - marketing of the products
     - delivery of the products to the clients in the units
     - management of the pool of risks
   - health care provider delivers services as hospitalization, surgery, etc.

   See Figure

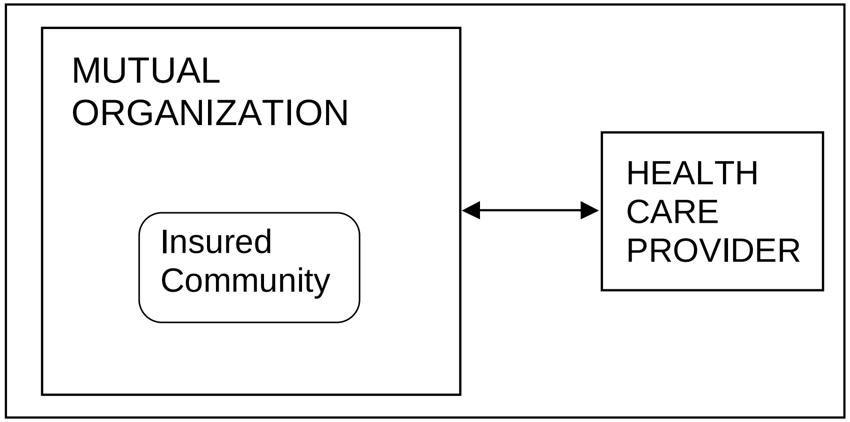
3. The arrangement relies on the HMIS only, which also acts as the health care provider (being, at the same time, responsible for all the operations listed under arrangement 2)

   See Figure

Health insurance products *(cont'd)*



HMIS-based health microinsurance arrangement (3)



Community-based health microinsurance arrangement

Different approach to the implementation adopted in the following arrangement

4. The arrangement relies on a *mutual organization*
   - The individuals who constitute the community are, at the same time, insured and involved in all the operations
   - An external institution acts as the health care provider

   See Figure

INTRODUCTION TO ACTUARIAL ASPECTS

SOME PRELIMINARY IDEAS

Actuarial aspects of health insurance modeling strictly related to:

- type of benefits, in particular as regards their definition in quantitative terms (fixed-amount, degree-related amount, expense reimbursement)
- policy term (one-year covers versus multi-year covers, and possibly lifelong covers)
- premium arrangement (single premium, natural premiums, level premiums, etc.)

To ease the presentation, refer to a single premium

The premium must rely on some "summary" of the random benefits

Benefits can consist, in general, of a sequence of random amounts paid throughout the policy duration $\Rightarrow$ we have to summarize:

1. with respect to time $\Rightarrow$ random *present value* of the benefits, referred at the time of policy issue
2. with respect to randomness $\Rightarrow$ calculating some typical values of the probability distribution of the random present value of the benefits, namely the *expected value*, the standard deviation, etc.

Step 1 requires the choice of the annual *interest rate*; in case of short policy duration (say, one year or even less) $\Rightarrow$ possible to skip this step

Step 2 requires appropriate *statistical bases* in order to construct the probability distribution of the random present value of the benefits, and the choice of typical values summarizing the distribution itself

Complexity of the statistical bases also depends on the sets of

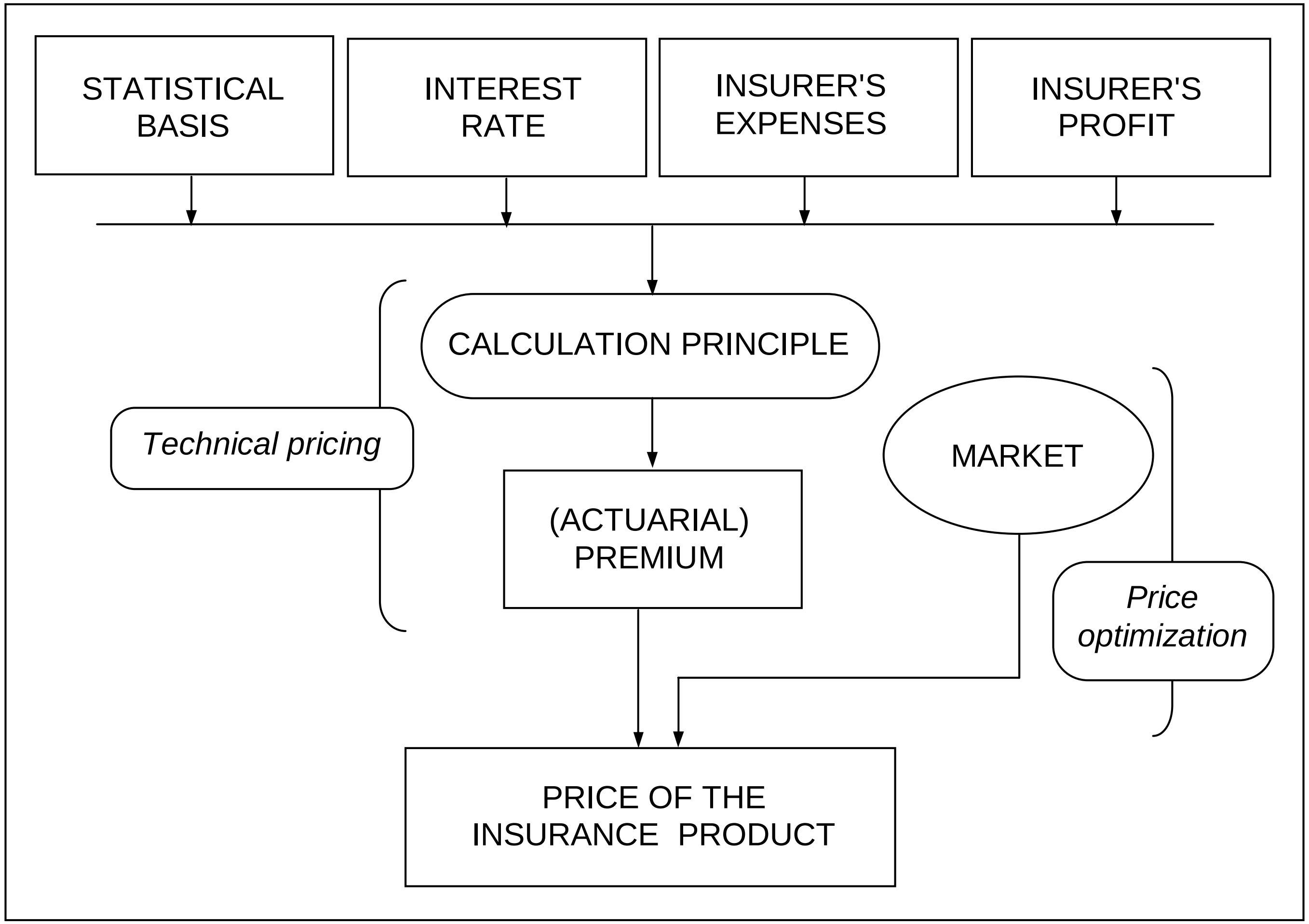
- individual *risk factors* accounted for in assessing the benefits (e.g. age, gender, health status, etc.)
- *rating factors* (among the risk factors) which are taken into account in premium calculation

Insurer's costs consisting in the payment of benefits are not the only items in premium calculation

Further items for premium calculations:

- ▷ *expenses* not directly connected with the amounts of benefits, for example general expenses $\Rightarrow$ *expense loading*
- ▷ *profit margin* (if not implicitly included via adjustment of the statistical bases), also including cost of capital ($\Rightarrow$ value creation)

See following Figure



Pricing an insurance product

Items listed above $\Rightarrow$ ingredients of a "recipe" called *premium calculation principle*

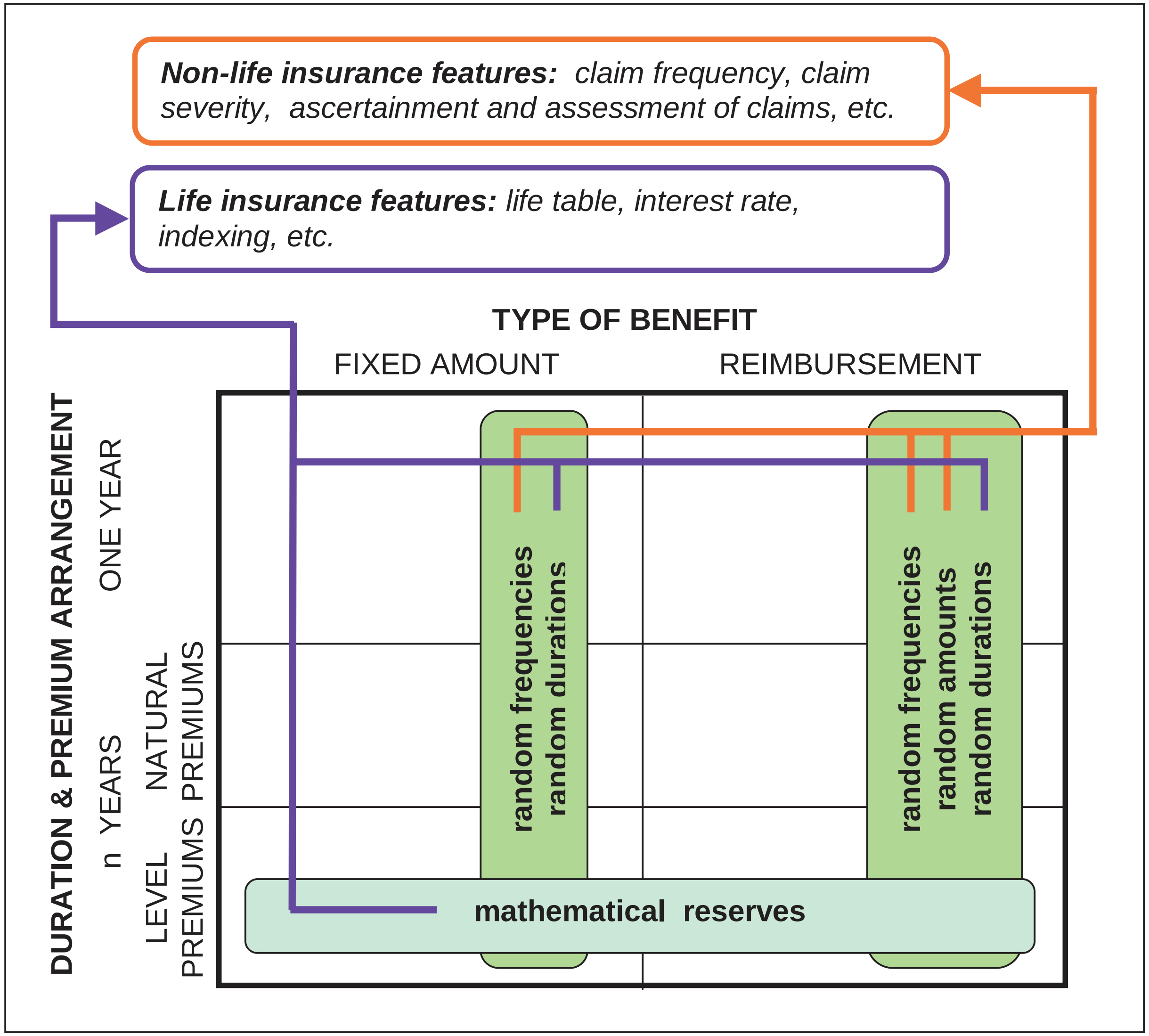
Output of premium calculation principle $\Rightarrow$ *actuarial premium*

Other items can intervene in determining the *price* of the product, e.g. competition on the market, clients' behavior, etc.

TECHNICAL FEATURES OF PREMIUM CALCULATION

Pricing health insurance products $\Rightarrow$ a mix of life insurance and non-life insurance technical tools

Either "life" or "non-life" aspects prevailing according to type of benefits, policy term, premium arrangement, etc.



Life and non-life technical features of health insurance products

Life insurance aspects

Mainly refer to medium term and long term contracts: disability annuities, LTC insurance, some types of sickness covers

- Survival modeling

  benefits are due in case of life $\Rightarrow$ survival probabilities should not be underestimated

- Financial issues

  asset accumulation (backing technical reserves), return to policyholders

Non-Life insurance aspects

- Claim frequency relates to all types of covers

  problems: availability, data format, experience monitoring and experience rating
- Claim size concerns insurance covers providing
  - ▷ expense-related benefits (e.g. reimbursement of medical expenses)
  - ▷ degree-related benefits (e.g. degree of disability)
- Expenses
  - ▷ Ascertainment and assessment of claims
  - ▷ Checking the health status in case of non-necessarily permanent disability

INTRODUCTION

Focus on insurance products which provide:

1. a fixed daily benefit in the case of (short-term) disability
2. a hospitalization benefit, that is, a fixed daily benefit during hospital stays
3. medical expenses reimbursement

Products of type 1 and 2 have similar technical features $\Rightarrow$ we can simply refer to both of them under the label “fixed daily benefit”

We first address insurance products with a one-year cover period, then we move to products providing a multi-year cover

One-year covers

Calculations for one-year covers have "non-life" technical features

These features are combined with "life" insurance characteristics in multi-year sickness insurance covers

Notation and assumptions

N = random number of claims for the generic insured, within the one-year cover period, with possible outcomes $0, 1, 2, \dots$; the number N is also called random *claim frequency*

X_j = random amount of the insured's j-th claim (e.g. medical expenses)

Y_j = insurer's random payment for the j-th claim, called random *claim amount* or *claim severity*, given by a function of X_j, such that $Y_j \leq X_j$, reflecting the policy conditions, i.e. deductible, limit value, etc. (e.g. reimbursement of medical expenses)

S = random total annual payment to the generic insured, or random *aggregate claim amount*:

$$S = \begin{cases} 0 & \text{if } N = 0 \\ Y_1 + Y_2 + \cdots + Y_N & \text{if } N > 0 \end{cases}$$

Premium calculation: *equivalence principle*

Equivalence premium given by the expected value of the total annual payment to the generic insured:

$$\Pi = \mathbb{E}[S]$$

To approximately take into account the timing of payments:

$$\Pi = \mathbb{E}[S]\,(1+i)^{-\frac{1}{2}}$$

where i = the interest rate

Assumptions usually accepted for calculation of $\mathbb{E}[S]$:

1. the random variables $X_1, X_2, \ldots, X_n$ are independent of the random number N
2. whatever the outcome n of N, the random variables $X_1, X_2, \ldots, X_n$ are
   (a) mutually independent
   (b) identically distributed, and hence with a common expected value, say $\mathbb{E}[X_1]$

Further assume:

3. same policy conditions applied to all the claims, that is, $Y_j = \phi(X_j)$ for $j = 1, 2, \ldots, n$; $\Rightarrow$ $Y_1, Y_2, \ldots, Y_n$ are identically distributed with common expected value, say, $\mathbb{E}[Y_1]$

Thanks to above assumptions:

$$\mathbb{E}[S] = \mathbb{E}[Y_1]\,\mathbb{E}[N]$$

Quantities $\mathbb{E}[Y_1]$ (expected claim severity) and $\mathbb{E}[N]$ (expected claim frequency), and interest rate i: *technical basis* for premium calculation

From equivalence premiums to gross premiums

From previous Equations $\Rightarrow$ random profit from the generic policy, $\Pi - S$, has expected value equal to zero:

$$\mathbb{E}[\Pi - S] = \Pi - \mathbb{E}[S] = 0$$

In contrast with a reasonable profit target

Further, expenses pertaining to the policy, as well as general expenses related to the portfolio, not accounted for

Actually, premiums paid by policyholders are *gross premiums* (also called *office premiums*), rather than equivalence premiums

Gross premiums determined from equivalence premiums by adding:

1. profit loading and contingency margins facing the risk that claims (and possibly expenses) are higher than expected
2. expense loading, meeting various insurer's expenses

Items 1: *profit/safety loading*

- ▷ if claim (and expense) actual experience within the portfolio coincides with the related expectation $\Rightarrow$ items 1 contribute to the portfolio profit
- ▷ in the case of experience worse than expectation $\Rightarrow$ items 1 lower the probability (and the severity) of possible portfolio losses

Adding profit/safety loading to the equivalence premium $\Rightarrow$ *net premium*

For example, profit/safety loading = fixed percentage of the equivalence premium

Whatever the formula $\Rightarrow$ *explicit* profit/safety loading

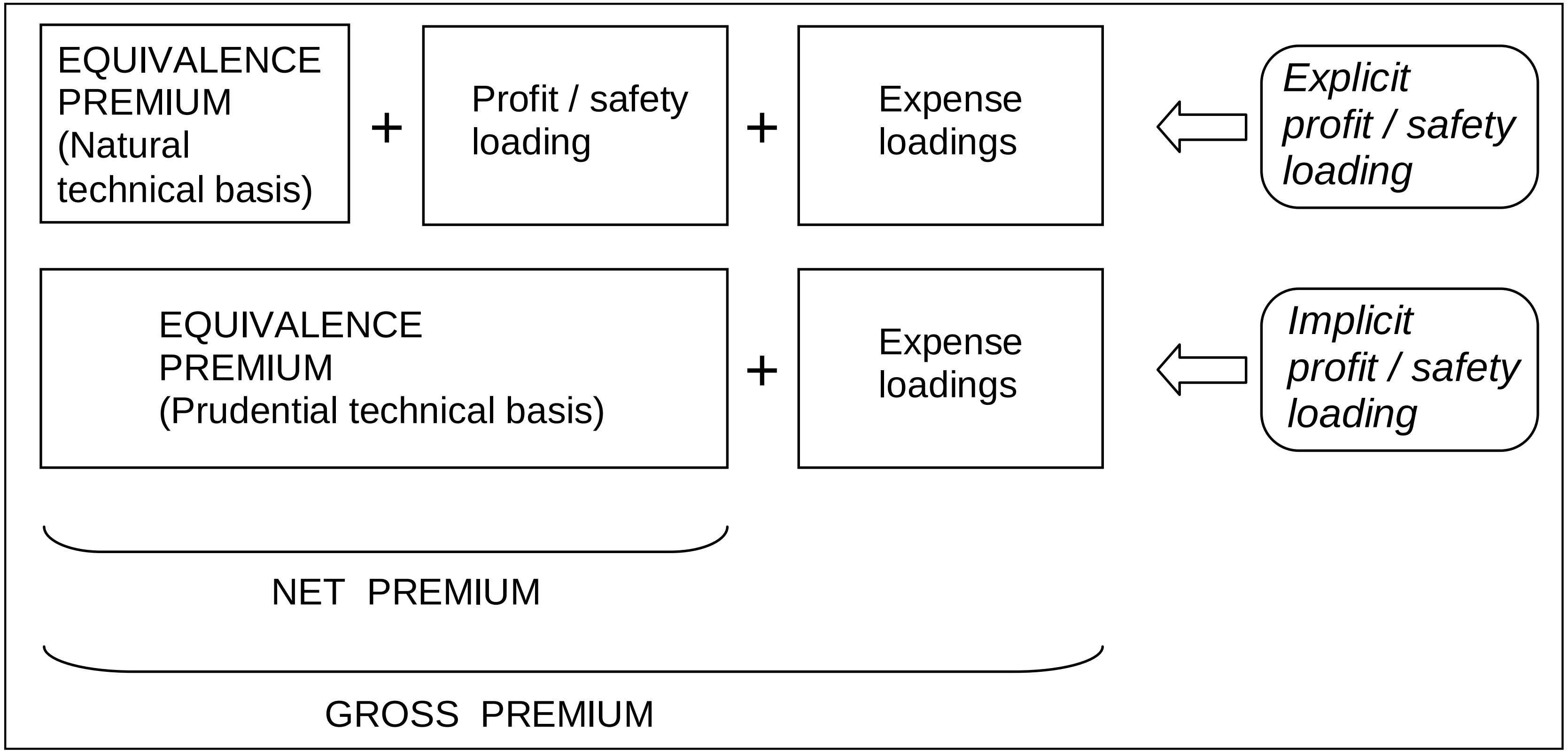
This approach relies on the use of a *natural technical basis* for equivalence premium calculation $\Rightarrow$ basis which provides a realistic description of claim (and interest) scenario; also referred to as the *second-order technical basis*

Equivalence principle can also be implemented by adopting a *prudential technical basis* (or *safe-side technical basis* $\Rightarrow$ profit/safety loading already included in the equivalence premium, then coinciding with the net premium

Basis also referred to as *first-order technical basis* $\Rightarrow$ *implicit* profit/safety loading

Prudential technical basis: expected claim frequency and expected claim severity worse than those realistically expected

Implicit loading approach much more common in life insurance



Premium components

Statistical estimation

Focus on quantities which can be used to estimate:

- ▷ expected claim frequency, $\mathbb{E}[N]$
- ▷ expected claim severity, $\mathbb{E}[Y_1]$
- ▷ expected total payout for a policy, $\mathbb{E}[S]$

Refer to a homogeneous portfolio, consisting of r insured risks, all issued at the same time and all with term one year

Homogeneity $\Rightarrow$ policies similar in respect of:

- type of risk covered (e.g. either medical expense reimbursement, or fixed daily benefit)
- policy conditions (deductibles, limit values)
- propensity to incur into a claim
- possible severity of a claim, etc.

Portfolio of policies providing *medical expense reimbursement*

- z = number of claims in total ($z \lesseqgtr r$) during the year
- claim amounts $y_1, y_2, \ldots, y_z$

$\Rightarrow$ aggregate information (we do not know which policies have reported claims)

Claim amount per policy:

$$Q = \frac{y_1 + y_2 + \cdots + y_z}{r}$$

also called *risk premium*, or *average claim cost*

If $\Pi = Q \;\Rightarrow$ insurer's on balance

$\Rightarrow$ Q looked at as *observed premium*

Quantity $Q \;\Rightarrow\;$ estimate of $\mathbb{E}[S]$

Split Q as follows:

Average number of claims per policy, or *average claim frequency*

$$\bar{n} = \frac{z}{r}$$

Average claim amount per claim, or *average claim severity*

$$\bar{y} = \frac{y_1 + y_2 + \cdots + y_z}{z}$$

In particular:

- $\bar{n} \;\Rightarrow\;$ estimate of $\mathbb{E}[N]$
- $\bar{y} \;\Rightarrow\;$ estimate of $\mathbb{E}[Y_1]$

Then:

$$Q = \bar{y}\,\bar{n}$$

Portfolio of *fixed daily benefit policies*

- ▷ z = number of claims in total ($z \lesseqgtr r$) during the year
- ▷ $d_1, d_2, \ldots, d_z$ lengths (in days) of the z claims

Average length of claim per policy, also called *morbidity coefficient*:

$$\mu = \frac{d_1 + d_2 + \cdots + d_z}{r}$$

Assume the same daily benefit b for all the insureds $\Rightarrow$ average claim amount per policy:

$$Q = b\,\frac{d_1 + d_2 + \cdots + d_z}{r} = b\,\mu$$

Average length per claim:

$$\bar{d} = \frac{d_1 + d_2 + \cdots + d_z}{z}$$

As the average number of claims per policy $= \bar{n} = \dfrac{z}{r}$, we find:

$$\mu = \bar{d}\,\bar{n}$$

Amount Q split as follows:

$$Q = b\,\mu = b\,\bar{d}\,\bar{n}$$

In particular:

- $\bar{n} \;\Rightarrow\;$ estimate of $\mathbb{E}[N]$
- $b\,\bar{d} \;\Rightarrow\;$ estimate of $\mathbb{E}[Y_1]$

Remark

More general (and realistic) setting:

- ▷ amounts exposed to risk (determined by allowing for deductibles and limit values), if policies providing medical expenses reimbursement are concerned
- ▷ the exposure time (within one observation year)

Risk factors and rating classes

Individual *risk factors* (age, gender, current health conditions, occupation, etc.) $\Rightarrow$ should be taken into account when estimating quantities (average claim frequency, average claim amount per claim)

Classification

- ▷ *Objective* risk factors: physical characteristics of the insured, in particular: age, gender, health records, occupation
- ▷ *Subjective* risk factors: personal attitude towards health $\Rightarrow$ individual demand for medical treatments and application for insurance benefits

Another classification

- *Observable* risk factors: factors whose impact on claim frequency and claim severity can be assessed during the underwriting phase; examples: age, gender, occupation, etc. Objective risk factors are usually observable factors
- *Non-observable* risk factors (at least at the time of policy issue); examples: personal attitude towards health, objective individual frailty (although related information can be drawn from the insured's health records)

Among the objective and observable risk factors: age

See Table

Actuarial models for sickness insurance *(cont'd)*

x	$100\ \bar{n}_x$	x	$100\ \bar{n}_x$
$15-19$	6.54	$45-49$	11.17
$20-24$	7.13	$50-54$	12.35
$25-29$	5.72	$55-59$	18.71
$30-34$	5.71	$60-64$	19.62
$35-39$	6.23	$65-69$	24.90
$40-44$	10.03		
$100\ \bar{n} = 10.48$			

Average claim frequency (Source: ISTAT)

Accounting for all the observable risk factors $\Rightarrow$ split a population (for example, potential policyholders) into *risk classes*

Resulting premium rating structure could be considered too complex, or some premium rates too high

Some risk factors could not be admitted by insurance regulation

A first "simplification" obtained disregarding one or more risk factors

Two or more risk classes aggregated into one *rating class* $\Rightarrow$ some insureds pay a premium higher than their "true" premium (resulting from the risk classification), while other insureds pay a premium lower than their "true" premium

Equilibrium inside a rating class relies on a money transfer among individuals belonging to different risk classes $\Rightarrow$ *solidarity* (among the insureds)

Premium calculation

Account for age only, as a risk factor

Assume:

$$\bar{y}_x,\ \bar{n}_x,\ \bar{d}_x$$

estimated for any integer age x, $x \in [x_{\min}, x_{\max}]$ (the insurable age range)

For a medical expense reimbursement policy:

$$\Pi_x = \bar{y}_x\, \bar{n}_x\, (1+i)^{-\frac{1}{2}}$$

For a daily benefit b:

$$\Pi_x = b\, \bar{d}_x\, \bar{n}_x\, (1+i)^{-\frac{1}{2}}$$

Considering just the average claim frequency as a function of the age:

$$\Pi_x = \bar{y}\,\bar{n}_x\,(1+i)^{-\frac{1}{2}}$$

$$\Pi_x = b\,\bar{d}\,\bar{n}_x\,(1+i)^{-\frac{1}{2}}$$

where $\bar{y}$ and $\bar{d}$ represents overall averages

Factorize $\bar{n}_x$, $\bar{y}_x$ and $\bar{d}_x$, according to the logic of a *multiplicative model*:

$$\bar{n}_x = \bar{n}\,t_x$$

$$\bar{y}_x = \bar{y}\,u_x$$

$$\bar{d}_x = \bar{d}\,v_x$$

where $\bar{n}$, $\bar{y}$ and $\bar{d}$ do not depend on age, whereas *ageing coefficients* t_x, u_x, and v_x express the impact of age as a risk factor

If specific age effect does not change throughout time, claim monitoring can be restricted to $\bar{n}$, $\bar{y}$, $\bar{d}$ observed over the whole portfolio $\Rightarrow$ more reliable estimates

Example

Consider a policy which provides a daily benefit $b = 100$

Assume $i = 0.02$ and the following statistical basis (graduation of ISTAT data):

$$\bar{n}_x = \bar{n}\, t_x = 0.1048 \times \left(0.272859 \times \mathrm{e}^{0.029841\, x}\right)$$

$$\bar{d}_x = \bar{d}\, v_x = 10.91 \times \left(0.655419 \times \mathrm{e}^{0.008796\, x}\right)$$

Average claim frequency, average claim duration and premium shown in following Table for various ages

Actuarial models for sickness insurance *(cont'd)*

x	$\bar{n}_x$	$\bar{d}_x$	Π_x
30	0.07000	9.30991	64.53
35	0.08126	9.72849	78.28
40	0.09434	10.16590	94.96
45	0.10952	10.62298	115.20
50	0.12714	11.10060	139.74
55	0.14760	11.59970	169.53
60	0.17135	12.12124	205.65
65	0.19892	12.66623	249.48
70	0.23093	13.23572	302.64

Average claim frequency, average claim duration and premium

MULTI-YEAR COVERS

Focus on multi-year non-cancellable policies, conditions stated at issue and cannot be changed throughout the whole policy duration

Some preliminary ideas

A multi-year cover can be financed, in particular, via:

1. single premium
2. natural premiums
3. level premiums (throughout the whole policy duration).

Other possible premium arrangements:

4. "shortened" level premiums (i.e. level premiums payable throughout a period shorter than the policy duration)
5. stepwise level premiums

Focus on arrangements 1 to 3 only

Arrangement 2 (natural premiums) $\Rightarrow$ technical equilibrium on an annual basis $\Rightarrow$ no policy reserve required (but the premium reserve, or reserve for unearned premiums)

Arrangements 1 and 3 $\Rightarrow$ technical equilibrium only on the total policy duration as a whole $\Rightarrow$ policy reserve has to be maintained

What about the policy reserve in the case the insured stops premium payment and withdraws from the contract ?

Possible policy conditions:

1. amount of reserve is paid-out to the policyholder $\Rightarrow$ the reserve is *transferable* (e.g. to a new sickness insurance contract) $\Rightarrow$ the policyholder can *surrender* the contract
2. amount of reserve is retained by the insurer, and can be shared, according to a cross-subsidy principle, among the policies still in-force $\Rightarrow$ a policy *lapsation* simply occurs

Condition 1 $\Rightarrow$ the reserve constitutes a *nonforfeiture benefit*, as the amount will not be lost because of premature cessation of premium payment

Condition 2 $\Rightarrow$ the reserve is shared among the policies still in-force $\Rightarrow$ cross-subsidy mechanism similar to the mutuality mechanism which works because of mortality among insureds

Actuarial perspective $\Rightarrow$ different probabilistic structures needed in the two cases, for premium and reserve calculations

Transferable reserve $\Rightarrow$ usual survival probability required in the calculations

Non-transferable reserve (retained by the insurer) $\Rightarrow$ the probability that the policy is in-force needed, as both mortality and lapses must be accounted for

In what follows: assume transferable reserves

Premiums

Focus on policies providing either medical expense reimbursement or a fixed daily benefit

Notation (time in years):

x = insured's age at policy issue (time $t = 0$)

m = policy term

${}_hp_x$ = probability, for a person age x, of being alive at age $x + h$

$\mathit{\Pi}_{x,m}$ = actuarial value of benefits at time $t = 0$

$$\mathit{\Pi}_{x,m} = \sum_{h=0}^{m-1} {}_hp_x \, (1+i)^{-h} \, \mathit{\Pi}_{x+h}$$

where $\mathit{\Pi}_{x+h}$ is given by previous equations

Equivalence principle $\Rightarrow$ *single premium* = actuarial value of benefits

Remark

> Sickness benefits are *living benefits*, that is, benefits are payable as long as the insured is alive (and sick) $\Rightarrow$ safe-side assessment of the insurer's liabilities requires that the insureds' mortality should not be overestimated

Quantities $\Pi_x, \Pi_{x+1}, \dots, \Pi_{x+m-1}$: *natural premiums* of the m-year insurance cover

Usually, we find:

$$\Pi_x < \Pi_{x+1} < \cdots < \Pi_{x+m-1}$$

because of the age effect (see previous Table)

Single premium in a multiplicative model; if

$$\Pi_x = \bar{y}_x \, \bar{n}_x \, (1+i)^{-\frac{1}{2}} = \bar{y} \, \bar{n} \, u_x \, t_x \, (1+i)^{-\frac{1}{2}}$$

then:

$$\Pi_{x,m} = \sum_{h=0}^{m-1} {}_hp_x \, (1+i)^{-h} \, \bar{y}_{x+h} \, \bar{n}_{x+h} \, (1+i)^{-\frac{1}{2}}$$

$$= \bar{y} \, \bar{n} \sum_{h=0}^{m-1} {}_hp_x \, (1+i)^{-h-\frac{1}{2}} \, u_{x+h} \, t_{x+h}$$

and briefly:

$$\Pi_{x,m} = K \sum_{h=0}^{m-1} w_{x,h} = K \, \pi_{x,m}$$

Various premium arrangements based on sequence of periodic premiums

In particular: sequence of natural premiums

$$\Pi_x, \Pi_{x+1}, \ldots, \Pi_{x+m-1}$$

$\Rightarrow$ implies an increasing annual cost to the policyholder (see inequalities)

To avoid increasing costs $\Rightarrow$ *annual level premiums*

Assuming annual level premiums, $P_{x,m}$, payable for m years:

$$P_{x,m} = \frac{\Pi_{x,m}}{\ddot{a}_{x:\overline{m}|}}$$

where $\ddot{a}_{x:\overline{m}|}$ = actuarial value of a unitary temporary life annuity payable at the beginning of each year

We find:

$$P_{x,m} = \frac{\sum_{h=0}^{m-1} {}_hp_x \, (1+i)^{-h} \, \mathit{\Pi}_{x+h}}{\sum_{h=0}^{m-1} {}_hp_x \, (1+i)^{-h}}$$

$\Rightarrow$ annual level premium (assumed payable throughout the whole policy duration) expressed as the arithmetic weighted average of the natural premiums

Example

Refer to fixed daily benefits. Assume:

$i = 0.02$

$b = 100$

$\bar{n}_y$, $\bar{d}_y$ as in previous Example

mortality assumption expressed by the first Heligman - Pollard law

See following Tables and Figures

First Heligman-Pollard law:

$$\frac{q_x}{1-q_x} = a^{(x+b)^c} + \, d\,\mathrm{e}^{-e\,(\ln x - \ln f)^2} + g\,h^x$$

a	b	c	d	e	f	g	h
0.00054	0.01700	0.10100	0.00013	10.72	18.67	1.464×10^{-5}	1.11

The first Heligman-Pollard law: parameters

$\overset{\circ}{e}_0$	$\overset{\circ}{e}_{40}$	$\overset{\circ}{e}_{65}$	Lexis
79.412	40.653	18.352	85

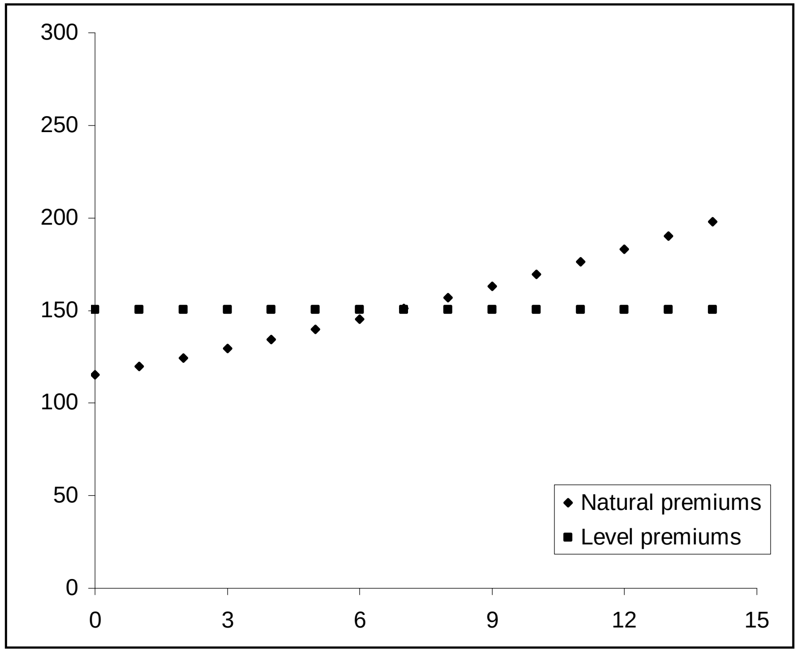
The first Heligman-Pollard law: some markers

x	$m=5$	$m=10$	$m=15$	$m=20$
30	334.86	701.78	1 103.13	1 540.82
35	406.02	850.13	1 334.46	1 859.98
40	492.11	1 028.79	1 611.12	2 237.62
45	596.11	1 242.92	1 938.80	2 676.86
50	721.35	1 497.42	2 320.53	3 172.86
55	871.42	1 795.66	2 752.71	–
60	1 049.76	2 136.79	–	–
65	1 258.68	–	–	–

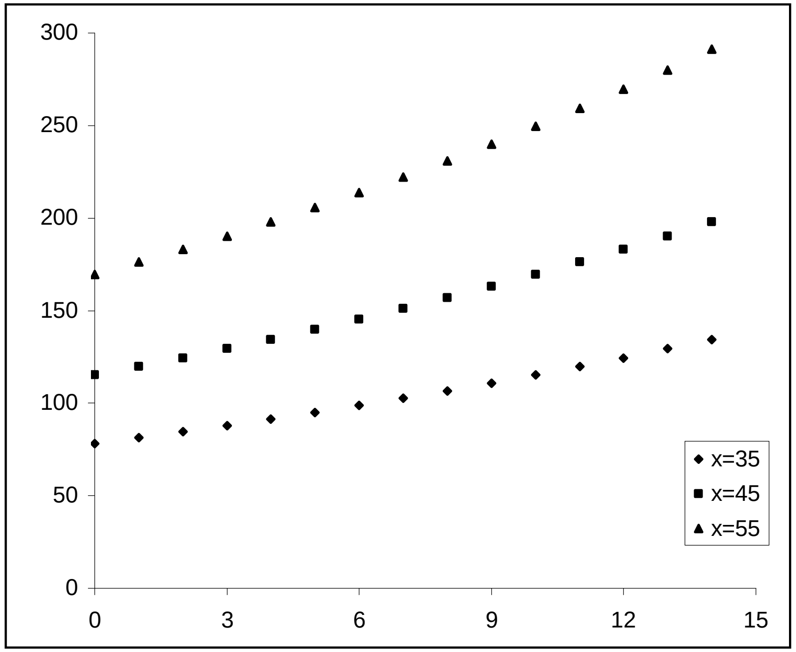
Single premiums

x	$m=5$	$m=10$	$m=15$	$m=20$
30	69.71	76.75	84.49	92.97
35	84.56	93.10	102.46	112.69
40	102.58	112.92	124.23	136.51
45	124.43	136.94	150.55	165.22
50	150.93	166.03	182.34	199.65
55	183.06	201.23	220.60	–
60	222.01	243.75	–	–
65	269.20	–	–	–

Annual level premiums



Natural premiums and annual level premium; $x = 45$, $m = 15$



Natural premiums for various ages at policy issue; $m = 15$

Reserves

Policy (prospective) reserve V_t, at (integer) time t:

actuarial value of future benefits - actuarial value of future premiums

In formal terms:

$$V_t = \mathrm{Ben}(t,m) - \mathrm{Prem}(t,m); \quad t = 0, 1, \dots, m$$

Reserve also referred to as *aging reserve*, or *senescence reserve*

Case of annual level premiums payable for the whole policy duration:

$$V_t = \Pi_{x+t,m-t} - P\, \ddot{a}_{x+t:\overline{m-t}|}; \quad t = 0, 1, \dots, m$$

where P = $P_{x,m}$

Of course:

$$V_0 = V_m = 0$$

We find:

$$V_t = \Pi_{x+t,1} - P + {}_1p_{x+t}\,(1+i)^{-1}\,(\Pi_{x+t+1,m-t-1} - P\,\ddot{a}_{x+t+1:\overline{m-t-1}|})$$

and, as $\Pi_{x+t,1} = \Pi_{x+t}$, we obtain the recursion:

$$V_t + P = \Pi_{x+t} + {}_1p_{x+t}\,(1+i)^{-1}\,V_{t+1}$$

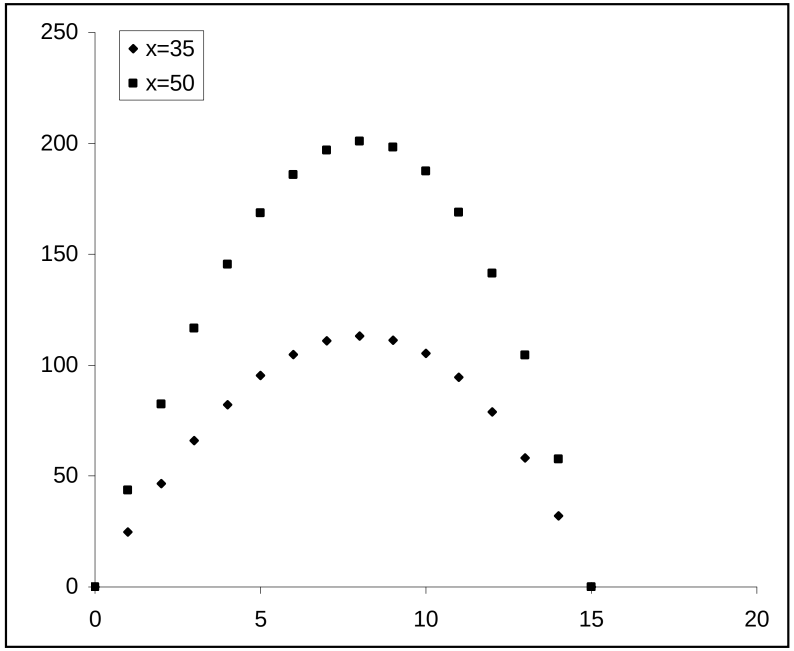
$\Rightarrow$ technical balance in year $(t, t+1)$.

In particular, interpretation:

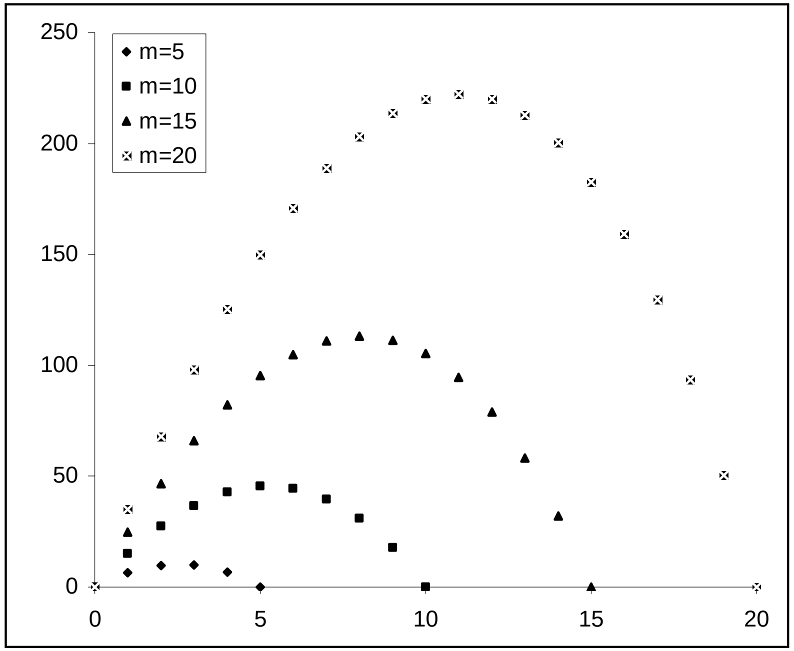
- V_t = amount of *assets* coming from the accumulation of part of premiums cashed before time t
- V_{t+1} = *debt* of the insurer for future benefits, net of the credit for future premiums

Example

Refer to fixed daily benefits. Data as in previous Example

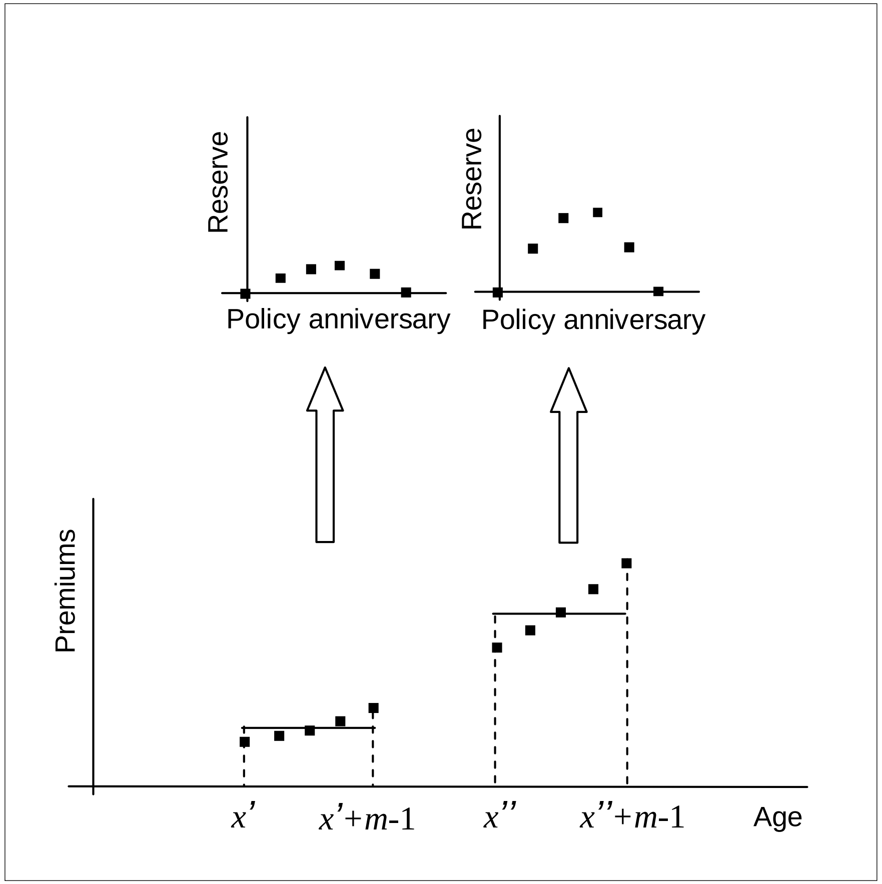


The policy reserve for two ages at policy issue; $m = 15$



The policy reserve for various policy terms; $x = 35$

Note: higher age at policy issue $\Rightarrow$ stronger increase in natural premiums $\Rightarrow$ higher reserve



Relation between natural premiums and reserve profile

Reserves at fractional durations

Reserve at fractional durations $\Rightarrow$ linear interpolation formulae, allowing for the unearned premium

Premium arrangement based on *natural premiums*

Premiums $\Pi_x, \Pi_{x+1}, \dots, \Pi_{x+m-1}$ at times $0, 1, \dots, m-1$ respectively

Reserve equal to zero at all the policy anniversaries, before cashing the premium $\Rightarrow$ $V_t = 0$ for $t = 0, 1, \dots, m-1$ (as well as $V_m = 0$)

Immediately after cashing the premium:

$$V_{t^+} = \Pi_{x+t}; \quad t = 0, 1, \dots, m-1$$

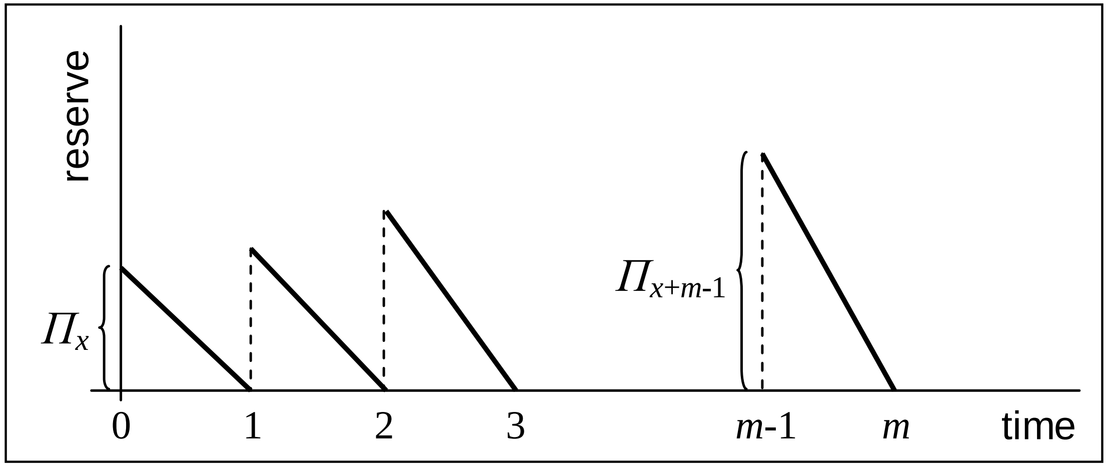
Premium used throughout the year to contribute to the payment of benefits to (same-age) policyholders, according to mutuality mechanism

Again, $V_{t+1} = 0$

At time $t+r$, with $t = 0, 1, \dots, m-1$ and $0 < r < 1$, let:

$$V_{t+r} = (1-r)\, V_{t^+} = (1-r)\, \Pi_{x+t}$$

Reserve $V_{t+r} = (1-r)\, \Pi_{x+t}$ = *unearned premium reserve*



Interpolated reserve profile in the case of natural premiums

Premium arrangement based on *annual level premiums*

After cashing the premium P at time t, the reserve raises from V_t to

$$V_{t^+} = V_t + P; \quad t = 0, 1, \ldots, m-1$$

Linear interpolation:

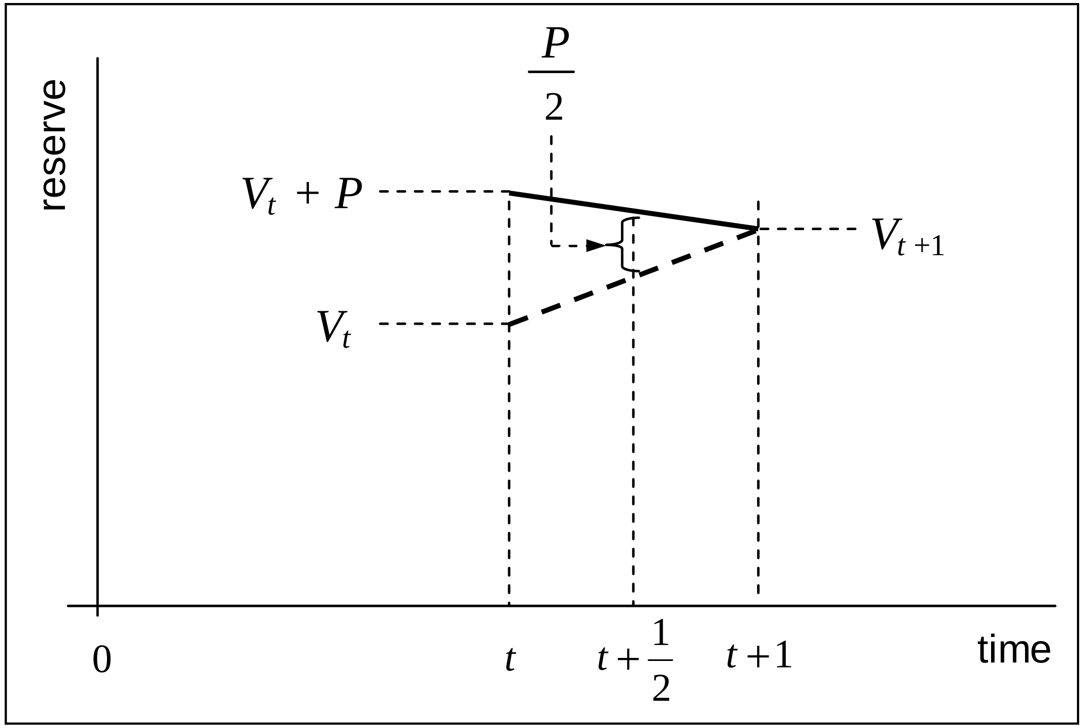
$$V_{t+r} = (1-r)\, V_{t^+} + r\, V_{t+1} = [(1-r)\, V_t + r\, V_{t+1}] + (1-r)\, P$$

See following Figure

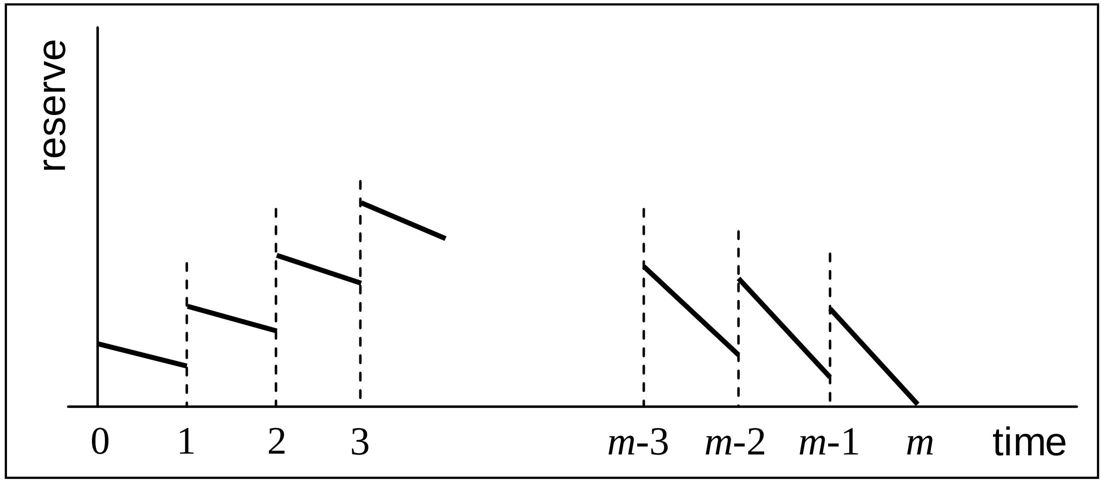
The term $(1-r)\, P$ represents the unearned premium reserve

Note, in particular:

- ▷ Interpolating between V_t (instead of V_{t^+}) and V_{t+1}
  $\Rightarrow$ underestimation of the reserve
- ▷ "Use" of premium P changes throughout time $\Rightarrow$ share of P used to cover sickness benefits according to the mutuality increasing throughout the policy duration; see Figure



Reserve interpolation in the case of annual level premiums



Interpolated reserve profile in the case of annual level premiums

Single premium arrangement

Single premium = $\Pi_{x,m}$

No jump in the reserve profile, but at the payment of the single premium

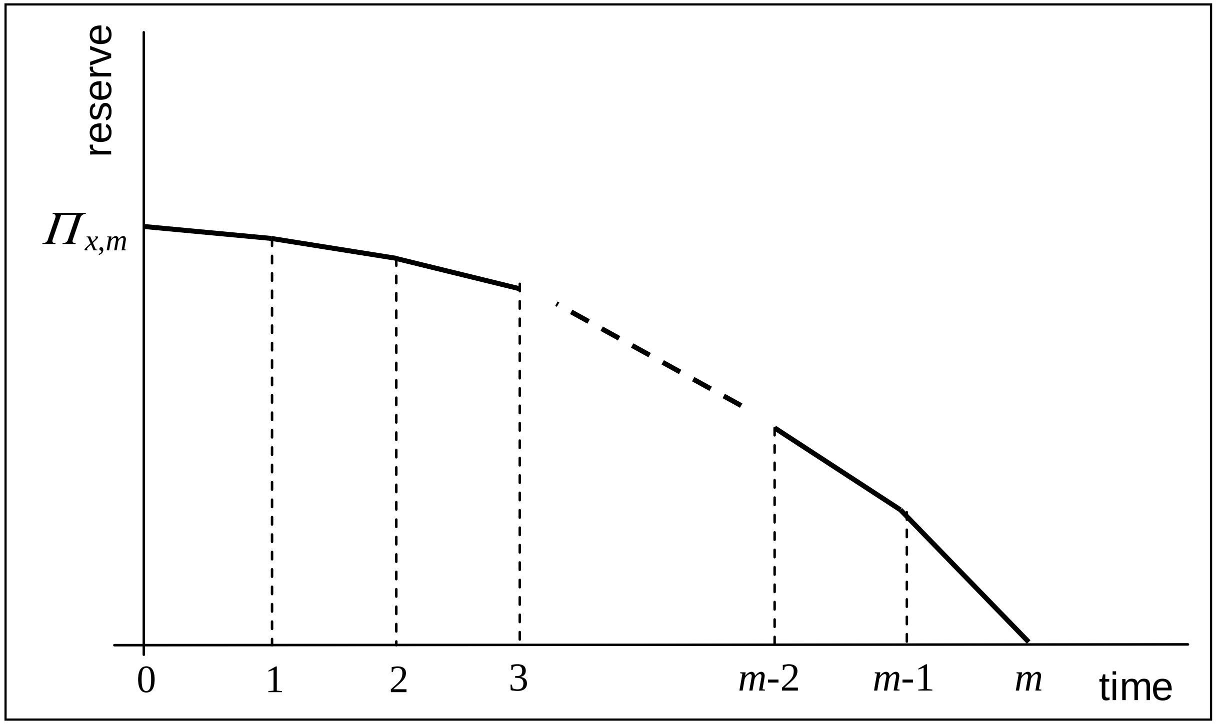
Reserve jump: $V_0 = 0 \;\rightarrow\; V_{0^+} = \Pi_{x,m}$

Linear interpolation:

$$V_r = (1-r)\,V_{0^+} + r\,V_1$$

$$V_{t+r} = (1-r)\,V_t + r\,V_{t+1} \text{ for } t = 1, 2, \dots, m-1$$

See following Figure



Interpolated reserve profile in the case of single premium

INDEXATION MECHANISMS

Refer to multi-year covers, exposed to the risk of significant changes in some scenario elements

Introduction

Possible changes, throughout the policy duration, in the elements accounted for at the time of policy issue, for example:

- the expected claim frequency
- the expected claim severity
- the money purchase power
- the mortality assumptions
- . . .

In case of changes:

- ▷ in the expected claim frequency or the expected claim severity $\Rightarrow$ technical equilibrium between premiums and benefits can be jeopardized
- ▷ in the money purchase power $\Rightarrow$ reduce effectiveness of fixed daily benefit

To avoid or limit consequences two basic approaches can be adopted

1. A forecast of future trend in some elements; of course, no guarantee can be provided as regards the effectiveness of such approach.
2. A periodic (e.g. yearly) *a posteriori* adjustment procedure can be adopted, first consisting in re-assessing the benefits according to the observed scenario, and then:
   - 2a. either re-determining future premiums and/or the reserve
   - 2b. or changing some policy conditions (raising the deductible, or lowering the limit values)

"Combined" solutions, based on approaches 1, 2a and 2b, can be implemented

In what follows, focus on solutions of type 2a

The adjustment model

Refer to an insurance policy with (initially) annual level premiums payable throughout the whole policy duration

At time t ($t = 1, 2, \ldots, m-1$) $\Rightarrow$ possible adjustment in future premiums and/or the reserve, due to a reassessment of the value of future benefits

Notation for quantities referred at time t before (possible) adjustment at that time, but including (possible) adjustments up to time $t-1$:

V_{t^-} = policy reserve

$\mathrm{Ben}(t^-, m)$ = actuarial value of future benefits

$\mathrm{Prem}(t^-, m)$ = the actuarial value of future premiums

Technical balance expressed by:

$$V_{t^-} + \mathrm{Prem}(t^-, m) = \mathrm{Ben}(t^-, m) \tag{1}$$

Assume at time t an increase in the actuarial value of future benefits at rate $j_t^{[\mathrm{B}]}$ ($j_t^{[\mathrm{B}]} > 0$)

To keep the balance $\Rightarrow$ quantities on the left-hand side of Eq. (1) must increase at rate $j_t^{[\mathrm{B}]}$

New balance condition:

$$\big(V_{t^-} + \mathrm{Prem}(t^-, m)\big)\,(1 + j_t^{[\mathrm{B}]}) = \mathrm{Ben}(t^-, m)\,(1 + j_t^{[\mathrm{B}]}) \tag{2}$$

Eq. (2) does not imply that both reserve and future premiums raise at rate $j_t^{[\mathrm{B}]}$; just the total value must be incremented at rate $j_t^{[\mathrm{B}]}$

Different rates can be adopted for reserve increment and future premiums increment, $j_t^{[\mathrm{V}]}$ and $j_t^{[\mathrm{P}]}$ respectively, provided that:

$$V_{t^-}\,(1+j_t^{[\mathrm{V}]}) + \mathrm{Prem}(t^-,m)\,(1+j_t^{[\mathrm{P}]}) = \mathrm{Ben}(t^-,m)\,(1+j_t^{[\mathrm{B}]}) \qquad (3)$$

As (1) must be fulfilled, Eq. (3) requires:

$$V_{t^-}\,j_t^{[\mathrm{V}]} + \mathrm{Prem}(t^-,m)\,j_t^{[\mathrm{P}]} = \mathrm{Ben}(t^-,m)\,j_t^{[\mathrm{B}]} \qquad (4)$$

Condition (4) $\Rightarrow$ reserve increment and future premiums increment have to balance the benefit value increment

Eq. (4) $\Rightarrow$ infinite solutions: given $j_t^{[\mathrm{B}]}$, unknowns are $j_t^{[\mathrm{V}]}$ and $j_t^{[\mathrm{P}]}$

Effectiveness of specific solutions $\Rightarrow$ see Example

Explicit expression for $j_t^{[\mathrm{B}]}$: from Eq. (4) we have

$$j_t^{[\mathrm{B}]} = \frac{V_{t^-}\, j_t^{[\mathrm{V}]} + \mathrm{Prem}(t^-, m)\, j_t^{[\mathrm{P}]}}{\mathrm{Ben}(t^-, m)}$$

and, replacing $\mathrm{Ben}(t^-, m)$ according to (1), we obtain:

$$j_t^{[\mathrm{B}]} = \frac{V_{t^-}\, j_t^{[\mathrm{V}]} + \mathrm{Prem}(t^-, m)\, j_t^{[\mathrm{P}]}}{V_{t^-} + \mathrm{Prem}(t^-, m)}$$

$\Rightarrow$ $j_t^{[\mathrm{B}]}$ = weighted arithmetic mean of $j_t^{[\mathrm{V}]}$ and $j_t^{[\mathrm{P}]}$; weights vary with past duration t

At any time:

if $j_t^{[\mathrm{V}]} < j_t^{[\mathrm{B}]}$, then $j_t^{[\mathrm{P}]} > j_t^{[\mathrm{B}]}$

if $j_t^{[\mathrm{P}]} < j_t^{[\mathrm{B}]}$, then $j_t^{[\mathrm{V}]} > j_t^{[\mathrm{B}]}$

Reserve at time t, after adjustment (but before cashing the premium):

$$V_t = V_{t^-}\,(1 + j_t^{[\mathrm{V}]})$$

Then:

$$V_t = \mathrm{Ben}(t^-, m)\,(1 + j_t^{[\mathrm{B}]}) - \mathrm{Prem}(t^-, m)\,(1 + j_t^{[\mathrm{P}]})$$

Let:

$$\mathrm{Ben}(t, m) = \mathrm{Ben}(t^-, m)\,(1 + j_t^{[\mathrm{B}]}) \qquad (5a)$$

$$\mathrm{Prem}(t, m) = \mathrm{Prem}(t^-, m)\,(1 + j_t^{[\mathrm{P}]}) \qquad (5b)$$

then:

$$V_t = \mathrm{Ben}(t, m) - \mathrm{Prem}(t, m)$$

$\Rightarrow$ V_t is the prospective reserve

Application to sickness insurance covers

Refer to policy

- issued at time 0, age x
- term m years
- benefit: either medical expense reimbursement or fixed daily benefit
- annual level premium $P_{x,m}$ (calculated at policy issue)

Premium:

$$P_{x,m} = \frac{\mathit{\Pi}_{x,m}}{\ddot{a}_{x:\overline{m}|}}$$

where:

$$\mathit{\Pi}_{x,m} = \sum_{h=0}^{m-1} {}_{h}p_x \, (1+i)^{-h} \, \mathit{\Pi}_{x+h}$$

At each anniversary $\Rightarrow$ possible benefit adjustment

Notation:

$$\text{Ben}(0, m) = \Pi_{x,m}$$

$$\text{Prem}(0, m) = P_{x,m}\, \ddot{a}_{x:m\rceil}$$

Premiums: let

$$P(0) = P_{x,m}$$

and then:

$$P(t) = P(t-1)\,(1 + j^{[\mathrm{P}]}(t)); \quad t = 1, 2, \ldots, m-1$$

Assume:

- ▷ actuarial value of benefits expressed by multiplicative model
- ▷ adjustment only concerns factor K (independent of age), and not age specificity

Let $K(0)$ = value at policy issue; then

$$\Pi_{x,m} = K(0)\,\pi_{x,m}$$

and:

$$K(t) = K(t-1)\,(1 + j^{[\mathrm{B}]}(t))$$

For a policy providing expense reimbursement:

$$K(0) = \bar{y}(0)\,\bar{n}$$

where $\bar{y}(0)$ = expected claim amount, assessed at policy issue

Assume that inflation affects claim amounts (constant expected frequency); we have:

$$K(t) = \bar{y}(t)\,\bar{n} = \bar{y}(t-1)\,(1 + j^{[\mathrm{B}]}(t))\,\bar{n}$$

with obvious meaning of $\bar{y}(t)$ and $\bar{y}(t-1)$

For a policy providing fixed daily benefit:

$$K(0) = b(0)\,\bar{d}\,\bar{n}$$

where $b(0)$ = initial benefit amount

Increase in benefit to keep the purchase power:

$$K(t) = b(t)\,\bar{d}\,\bar{n} = b(t-1)\,(1 + j^{[\mathrm{B}]}(t))\,\bar{d}\,\bar{n}$$

Whatever the type of benefit, note that:

$$\mathrm{Ben}(t^-, m) = K(t-1)\,\pi_{x+t,m-t}$$

$$\mathrm{Prem}(t^-, m) = P(t-1)\,\ddot{a}_{x+t:m-t\rceil}$$

$\Rightarrow$ Eqs. $(5a)$, $(5b)$ can be used to determine reserve V_t

Remark

In pratice:

- ▷ reserve increment (rate $j^{[\mathrm{V}]}(t)$) usually financed by the insurer through profit participation
- ▷ premium increment (rate $j^{[\mathrm{P}]}(t)$) paid by policyholders

Example

Policy providing medical expense reimbursement

- $x = 50,\ m = 15$
- annual premiums payable for the whole policy duration
- other data: see Examples on premiums and reserves

t	$j^{[B]}(t)$	$j^{[V]}(t)$	$j^{[P]}(t)$
1	0.00098	0.05	0
2	0.00198	0.05	0
3	0.00301	0.05	0
4	0.00407	0.05	0
5	0.00515	0.05	0
6	0.00625	0.05	0
7	0.00736	0.05	0
8	0.00850	0.05	0
9	0.00965	0.05	0
10	0.01081	0.05	0
11	0.01198	0.05	0
12	0.01316	0.05	0
13	0.01434	0.05	0
14	0.01552	0.05	0

Benefit adjustments (1)

t	$j^{[B]}(t)$	$j^{[V]}(t)$	$j^{[P]}(t)$
1	0.06	0	0.06120
2	0.06	0	0.06234
3	0.06	0	0.06345
4	0.06	0	0.06450
5	0.06	0	0.06550
6	0.06	0	0.06646
7	0.06	0	0.06737
8	0.06	0	0.06823
9	0.06	0	0.06905
10	0.06	0	0.06982
11	0.06	0	0.07055
12	0.06	0	0.07123
13	0.06	0	0.07187
14	0.06	0	0.07247

Benefit adjustments (2)

Remark 1

Sickness insurance covers are not “accumulation” products
$\Rightarrow$ amount of the mathematical reserve is relatively small (although the longer is the policy duration, the higher is the mathematical reserve; see numerical results in previous Examples)

Then:

- ▷ only increment of the reserve cannot maintain a significant raise in actuarial value of future benefits (see Table 1)
- ▷ on the contrary, raise in actuarial value of future benefits can be financed by reasonable increment of future premiums only (see Table 2)
- ▷ a longer policy term implies higher reserve amounts, and hence a more important role of the reserve increments in maintaining the raise in the actuarial value of future benefits

Remark 2

Indexing in lifelong sickness covers is analyzed in:

W. Vercruysse, J. Dhaene, M. Denuit, E. Pitacco and K. Antonio (2013), Premium indexing in lifelong health insurance

Available at:
`http://www.econ.kuleuven.ac.be/tew/academic/actuawet/pdfs/13-FJMS-SVIV-365.pdf`

LIFELONG COVERS

Actuarial value of the benefits provided by a lifelong sickness cover:

$$\Pi_{x,\infty} = \sum_{h=0}^{+\infty} {}_h p_x \, (1+i)^{-h} \, \Pi_{x+h}$$

Several periodic premium arrangements (besides natural premium arrangement):

1. lifelong level premiums
2. temporary level premiums
3. temporary stepwise level premiums

Arrangement 1:

$$P_{x,\infty(\infty)} = \frac{\Pi_{x,\infty}}{\ddot{a}_x}$$

Arrangement 2:

$$P_{x,\infty(r)} = \frac{\Pi_{x,\infty}}{\ddot{a}_{x:\overline{r}|}}$$

Arrangement 3 (for example):

$$P' \, \ddot{a}_{x:\overline{r'}|} + P'' \, {}_{r'|}\ddot{a}_{x:\overline{r''}|} + P''' \, {}_{r'+r''|}\ddot{a}_{x:\overline{r'''}|} = \Pi_{x,\infty}$$

A relation among P', P'', P''' must be assigned (reasonably, such that $P' < P'' < P'''$)

Some remarks

- Individual perspective: a lifelong sickness insurance policy provides the insured with appropriate coverage over his/her whole life $\Rightarrow$ a "high quality" insurance product

- Insurers' perspective: some problems may arise
  - ▷ sickness data related to very old ages may be scanty
  - ▷ need for forecasting mortality (and morbidity) over very long periods $\Rightarrow$ significant aggregate longevity risk
  - ▷ the longer the policy duration the higher is the reserve $\Rightarrow$ investment of assets backing the reserve gains in importance
  - ▷ higher reserve amounts $\Rightarrow$ more important role of the reserve itself in maintaining indexing mechanisms

ACTUARIAL MODELS FOR DISABILITY ANNUITIES

INTRODUCTION

In this Chapter:

- a simple probabilistic model
- calculation of actuarial values
- premiums
- reserves
- policy conditions
- allowing for disability-past-duration effect
- practical methods
- introduction to LTCI actuarial models

SOME PRELIMINARY IDEAS

Refer to insurance covers providing a disability annuity benefit b per annum when the insured is disabled, i.e. in state i

At policy issue the insured, aged x, is active, i.e. in state a

The policy term is m

The disability annuity is assumed to be payable up to the policy term m

For simplicity, assume that the benefit is paid at policy anniversaries (see following Figure)

This assumption is rather unrealistic, but the resulting model is simple and allows us to single out important basic ideas

No particular policy condition (e.g. deferred period, waiting period, etc.) is now considered

a	a	i	i	a	a	i	i	i	.....	state
	0	b	b	0	0	b	b	b	.....	benefit
0	1	2	3	4	5	6	7	8	.....	time

An example of disability annuity

Random present value Y of the benefit

$$Y = \sum_{h=1}^{m} B_h \, v^h$$

with v = annual discount factor and

$$B_h = \begin{cases} b & \text{if state} = i \\ 0 & \text{if state} \neq i \end{cases}$$

Expected present value (actuarial value)

$$\mathbb{E}[Y] = \sum_{h=1}^{m} \mathbb{E}[B_h]\, v^h$$

If x = age at policy issue, we have

$$\mathbb{E}[B_h] = b\ {}_h p_x^{ai}$$

where ${}_h p_x^{ai}$ = probability for an active individual age x of being disabled at age $x + h$, and then

$$\mathbb{E}[Y] = \sum_{h=1}^{m} b\ {}_h p_x^{ai}\, v^h$$

Practical difficulties in "directly" estimating le probabilities ${}_h p_x^{ai}$

Alternative approach needed

THE BASIC BIOMETRIC MODEL

Evolution of an insured risk throughout time $\Rightarrow$ sequence of events which determine cash flows of premiums and benefits

Logical support provided by multistate models

Multistate models for disability insurance

Disability insurance products $\Rightarrow$ relevant events are typically disablement, recovery and death

Evolution of a risk (an insured individual) then described in terms of the presence of the risk itself, at every point of time, in a certain *state*, belonging to a given set of states, or *state space*

Events correspond to *transitions* from one state to another state

Multistate model with

- *states*

active (or healthy):	state a
disabled (or invalid):	state i
dead:	state d

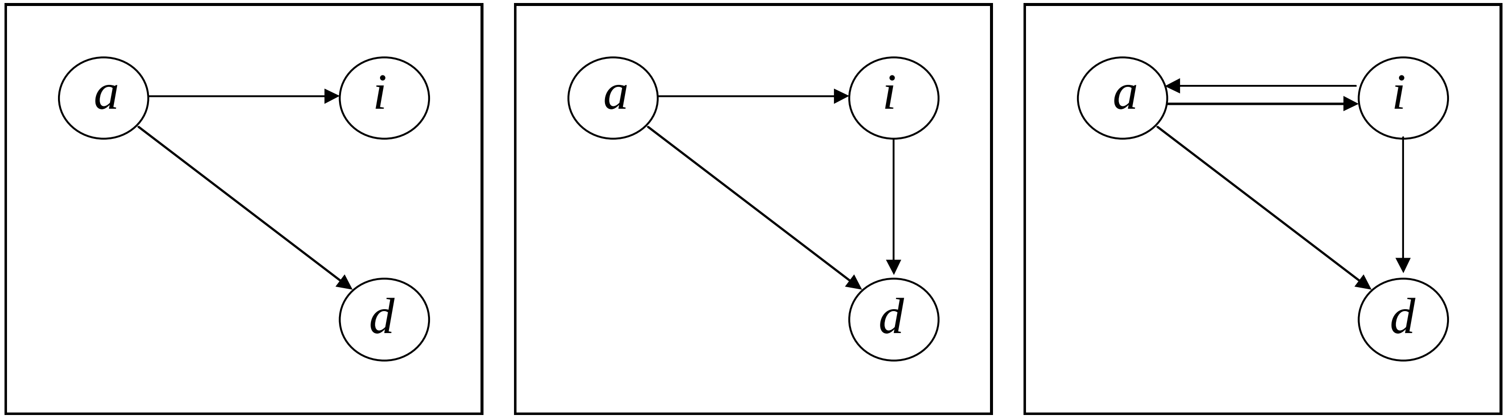
- *transitions* between states

disablement:	transition $a \to i$
death of an active:	transition $a \to d$
death of a disabled:	transition $i \to d$
recovery:	transition $i \to a$

Transitions actually considered depend on the particular type of benefits

See following Figures

Actuarial models for disability annuities *(cont'd)*



(a) (b) (c)

Three-state models

Model (a): lump sum in case of permanent disability

Model (b): annuity in case of permanent disability

Model (c): annuity in case of non-necessarily permanent disability

One-year transition probabilities

Approach:

- define a probabilistic model based on one-year transition probabilities (e.g. $p_y^{ai} = {}_1p_y^{ai}$), i.e. concerning state at age $y+1$ given the state at age y (conditional probabilities)
- use appropriate relations to obtain multi-year probabilities (e.g. ${}_hp_y^{ai};\ h = 2, 3, \dots$) from one-year probabilities

Usual assumption: no more than one transition can occur during one year, apart from possible death (see examples in the following Table)

State at age y	Transition(s)	State at age $y+1$	Allowed by the model ?
a	$\rightarrow$	i	yes
i	$\rightarrow$	a	yes
a	$\rightarrow i \rightarrow$	a	no
a	$\rightarrow i \rightarrow a \rightarrow$	i	no
a	$\rightarrow$	d	yes
i	$\rightarrow$	d	yes
a	$\rightarrow i \rightarrow$	d	yes
i	$\rightarrow a \rightarrow$	d	yes
a	$\rightarrow i \rightarrow a \rightarrow$	d	no

Examples of transitions between states

One-year transition probabilities related to an active insured age y:

$p_y^{aa} =$ probability of being active at age $y+1$

$q_y^{aa} =$ probability of dying within one year, death occurring in state a

$p_y^{ai} =$ probability of being disabled at age $y+1$

$q_y^{ai} =$ probability of dying within one year, death occurring in state i

Further:

$p_y^{a} =$ probability of being alive at age $y+1$

$q_y^{a} =$ probability of dying within one year

$w_y =$ probability of becoming disabled within one year

One-year transition probabilities related to a disabled insured age y:

$p_y^{ii} =$ probability of being disabled at age $y+1$

$q_y^{ii} =$ probability of dying within one year, death occurring in state i

$p_y^{ia} =$ probability of being active at age $y+1$

$q_y^{ia} =$ probability of dying within one year, death occurring in state a

Further:

$p_y^{i} =$ probability of being alive at age $y+1$

$q_y^{i} =$ probability of dying within one year

$r_y =$ probability of recovery within one year

Relations:

$$p_y^{aa} + p_y^{ai} = p_y^a$$

$$q_y^{aa} + q_y^{ai} = q_y^a$$

$$p_y^a + q_y^a = 1$$

$$p_y^{ai} + q_y^{ai} = w_y$$

$$p_y^{ia} + p_y^{ii} = p_y^i$$

$$q_y^{ii} + q_y^{ia} = q_y^i$$

$$p_y^i + q_y^i = 1$$

$$p_y^{ia} + q_y^{ia} = r_y$$

Thanks to assumption that no more than one transition can occur during one year (apart from possible death), p_y^{aa} and p_y^{ii} actually represent probabilities of remaining active and disabled respectively, from age y to $y+1$

If only permanent disability is allowed for:

$$p_y^{ia} = q_y^{ia} = 0$$

Remark

Hamza's notation:

- p $\Rightarrow$ probability of being alive
- q $\Rightarrow$ probability of dying
- 2 exponents $\Rightarrow$ state at age y (conditioning event), state at age $y+1$ or at death
- 1 exponent $\Rightarrow$ state at age y (conditioning event)

Only conditioning so far considered while defining the one-year probabilities: state occupied by the insured at age y
$\Rightarrow$ the probabilistic model is a *Markov chain*

Set of probabilities in the following Table: *stochastic matrix* (the sum of the items on each row is = 1), also called *transition matrix*, related to the Markov chain

	state at age $y+1$		
state at age y	a	i	d
a	p_y^{aa}	p_y^{ai}	q_y^a
i	p_y^{ia}	p_y^{ii}	q_y^i
d	0	0	1

Conditional probabilities of being is states a, i, d, at age $y+1$

Set of probabilities needed for actuarial calculations can be reduced by adopting *approximation formulae*

Common assumptions:

$$q_y^{ai} = w_y \, \frac{q_y^i}{2}$$

$$q_y^{ia} = r_y \, \frac{q_y^a}{2}$$

Hypotheses underpinning above formulae:

- uniform distribution of the first transition time within the year (the transition consisting in $a \rightarrow i$ or $i \rightarrow a$ respectively);
- the probability that the second transition ($i \rightarrow d$ or $a \rightarrow d$ respectively) occurs within the second half of the year is equal to one half of the probability that a transition of the same type occurs within the year

More rigorous approximations:

$$q_y^{ai} = w_y \ {}_{\frac{1}{2}}q^i_{y+\frac{1}{2}}$$

$$q_y^{ia} = r_y \ {}_{\frac{1}{2}}q^a_{y+\frac{1}{2}}$$

Thanks to q_y^{ai} approximation, and assuming that probabilities w_y, r_y, q_y^i and q_y^a (called *Zimmermann basic functions*) have been estimated, all other probabilities can be calculated; in particular:

$$p_y^{ai} = w_y - q_y^{ai} = w_y \left(1 - \frac{q_y^i}{2}\right)$$

$$p_y^{aa} = p_y^a - p_y^{ai} = p_y^a - w_y \left(1 - \frac{q_y^i}{2}\right)$$

$$q_y^{aa} = q_y^a - q_y^{ai} = q_y^a - w_y \, \frac{q_y^i}{2}$$

Multi-year transition probabilities

Notation, for example referring to an active insured age y:

$_hp_y^{aa} =$ probability of being active at age $y+h$

$_hp_y^{ai} =$ probability of being disabled at age $y+h$

etc.

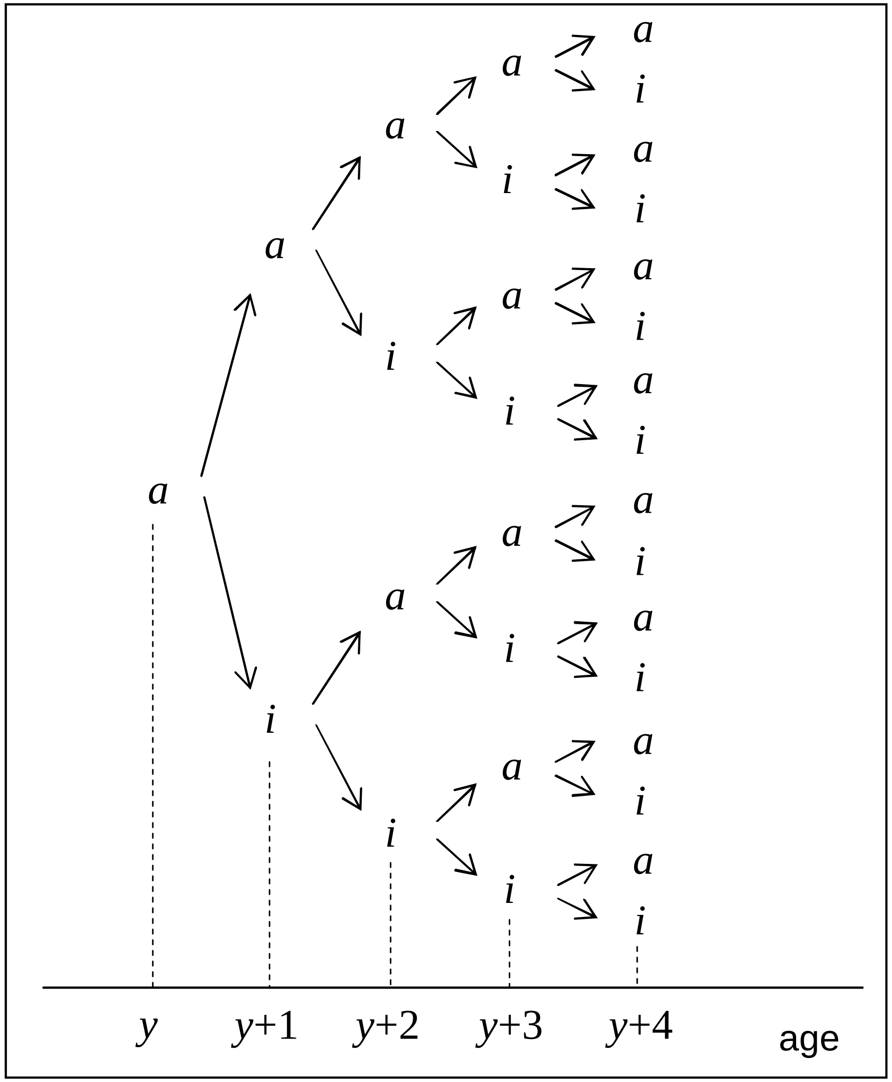
Example

Refer to a 4-year period

Following Figure: all possible $2^4 = 16$ disability stories (i.e. "paths") are plotted, which start from state a at age y and terminate either in state a or i at age $y+4$

Assume we have to calculate $_4p_y^{ai}$, given all the one-year probabilities p_{y+j}^{aa}, p_{y+j}^{ai}, p_{y+j}^{ia} and p_{y+j}^{ii}, for $j = 0, 1, 2, 3$

Actuarial models for disability annuities *(cont'd)*



Possible disability stories in a 4-year time interval

State i is reached, at age $y+4$, by 8 of the 16 paths

The 8 stories are mutually exclusive $\Rightarrow$ probability ${}_4p_y^{ai}$ equal to the sum of the probabilities of the 8 stories

For example, probability of the story

$$a \rightarrow a,\ a \rightarrow a,\ a \rightarrow a,\ a \rightarrow i$$

is expressed as follows:

$$p_y^{aa}\, p_{y+1}^{aa}\, p_{y+2}^{aa}\, p_{y+3}^{ai}$$

To eventually reach the state i $\Rightarrow$ final step must be either $a \rightarrow i$ or $i \rightarrow i$, depending on the state at age $y+3$

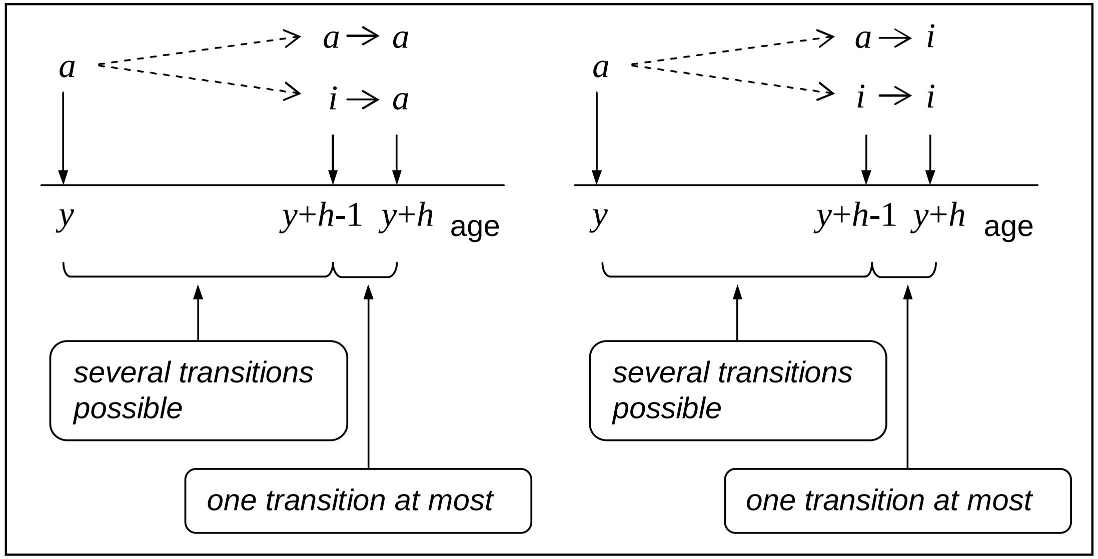
$\Rightarrow$ probability ${}_4p_y^{ai}$ can be expressed as follows:

$${}_4p_y^{ai} = {}_3p_y^{aa}\, p_{y+3}^{ai} + {}_3p_y^{ai}\, p_{y+3}^{ii}$$

Recurrent relationships (Chapman-Kolmogorov equations), for $h \geq 1$:

$$
\begin{aligned}
{}_h p_y^{aa} &= {}_{h-1}p_y^{aa}\, p_{y+h-1}^{aa} + {}_{h-1}p_y^{ai}\, p_{y+h-1}^{ia} \\
{}_h p_y^{ai} &= {}_{h-1}p_y^{aa}\, p_{y+h-1}^{ai} + {}_{h-1}p_y^{ai}\, p_{y+h-1}^{ii}
\end{aligned}
$$

with ${}_0 p_y^{aa} = 1$ and ${}_0 p_y^{ai} = 0$



Interpretation of recurrent relationships

Probabilities of remaining in a certain state for a given period: *occupancy probabilities*

For an individual aged y:

$_{h}p_{\overline{y}}^{\overline{aa}}$ = probability of remaining in state a for h years

$_{h}p_{\overline{y}}^{\overline{ii}}$ = probability of remaining in state i for h years

For $h = 1$:

$$_{1}p_{\overline{y}}^{\overline{aa}} = p_{y}^{aa}\,; \qquad _{1}p_{\overline{y}}^{\overline{ii}} = p_{y}^{ii}$$

In general, for $h \geq 1$:

$$_{h}p_{\overline{y}}^{\overline{aa}} = \prod_{k=0}^{h-1} p_{y+k}^{aa}\,; \qquad _{h}p_{\overline{y}}^{\overline{ii}} = \prod_{k=0}^{h-1} p_{y+k}^{ii}$$

Of course, $_{0}p_{\overline{y}}^{\overline{aa}} = {_{0}p_{\overline{y}}^{\overline{ii}}} = 1$

Example (cont)

Set of paths which eventually leading to the state i at age $y+4$ split into the 4 following subsets (see previous Figure):

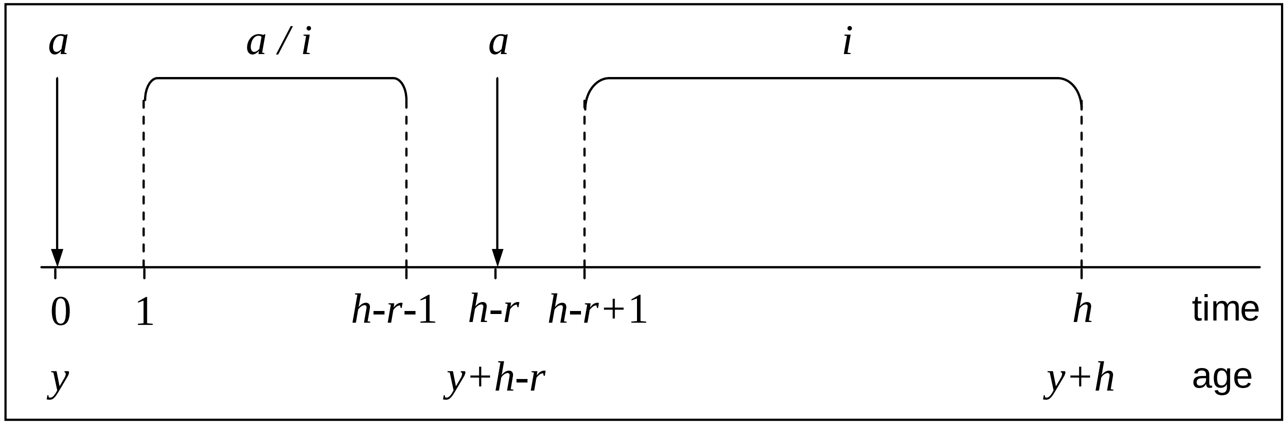
1. paths entering the state i between age $y+3$ and $y+4$
2. paths entering the state i between age $y+2$ and $y+3$, then remaining in i
3. the path entering the state i between age $y+1$ and $y+2$, then remaining in i
4. the path entering the state i between age y and $y+1$, then remaining in i

$\Rightarrow$ probability ${}_4p_y^{ai}$ expressed as follows:

$$ {}_4p_y^{ai} = \underbrace{{}_3p_y^{aa}\ p_{y+3}^{ai}}_{\text{subset } 1} + \underbrace{{}_2p_y^{aa}\ p_{y+2}^{ai}\ {}_1p_{y+3}^{\overline{ii}}}_{\text{subset } 2} + \underbrace{{}_1p_y^{aa}\ p_{y+1}^{ai}\ {}_2p_{y+2}^{\overline{ii}}}_{\text{subset } 3} + \underbrace{p_y^{ai}\ {}_3p_{y+1}^{\overline{ii}}}_{\text{subset } 4} $$

Relationship, involving the probability of remaining disabled:

$$ {}_{h}p_{y}^{ai} = \sum_{r=1}^{h} \left[{}_{h-r}p_{y}^{aa} \; p_{y+h-r}^{ai} \; {}_{r-1}p_{y+h-r+1}^{\overline{ii}} \right] \qquad (\circ) $$



Sequences of states and transitions: $a \rightarrow \; \ldots \; \rightarrow i$

ACTUARIAL VALUES

Refer to an individual active at age x (i.e. in state a)

Actuarial value, $a^{ai}_{x:\overline{m}|}$, of the disability insurance cover described above (the expected value of the random variable Y), with $b = 1$, is given by:

$$a^{ai}_{x:\overline{m}|} = \mathbb{E}[Y] = \sum_{h=1}^{m} v^h \; {}_hp^{ai}_x$$

Using Eq. $(^\circ)$:

$$a^{ai}_{x:\overline{m}|} = \sum_{h=1}^{m} v^h \sum_{r=1}^{h} \left[{}_{h-r}p^{aa}_x \; p^{ai}_{x+h-r} \; {}_{r-1}p^{\overline{ii}}_{x+h-r+1} \right]$$

Then, letting $j = h - r + 1$ and inverting the summation order:

$$a^{ai}_{x:m\rceil} = \sum_{j=1}^{m} {}_{j-1}p^{aa}_{x}\; p^{ai}_{x+j-1} \sum_{h=j}^{m} v^{h}\; {}_{h-j}p^{\overline{ii}}_{x+j}$$

Quantity

$$\ddot{a}^{i}_{x+j:m-j+1\rceil} = \sum_{h=j}^{m} v^{h-j}\; {}_{h-j}p^{\overline{ii}}_{x+j}$$

= actuarial value of a temporary immediate annuity paid to a disabled insured aged $x + j$ while he/she stays in state i, up to the end of the policy term m, briefly actuarial value of a *disability annuity*

We obtain:

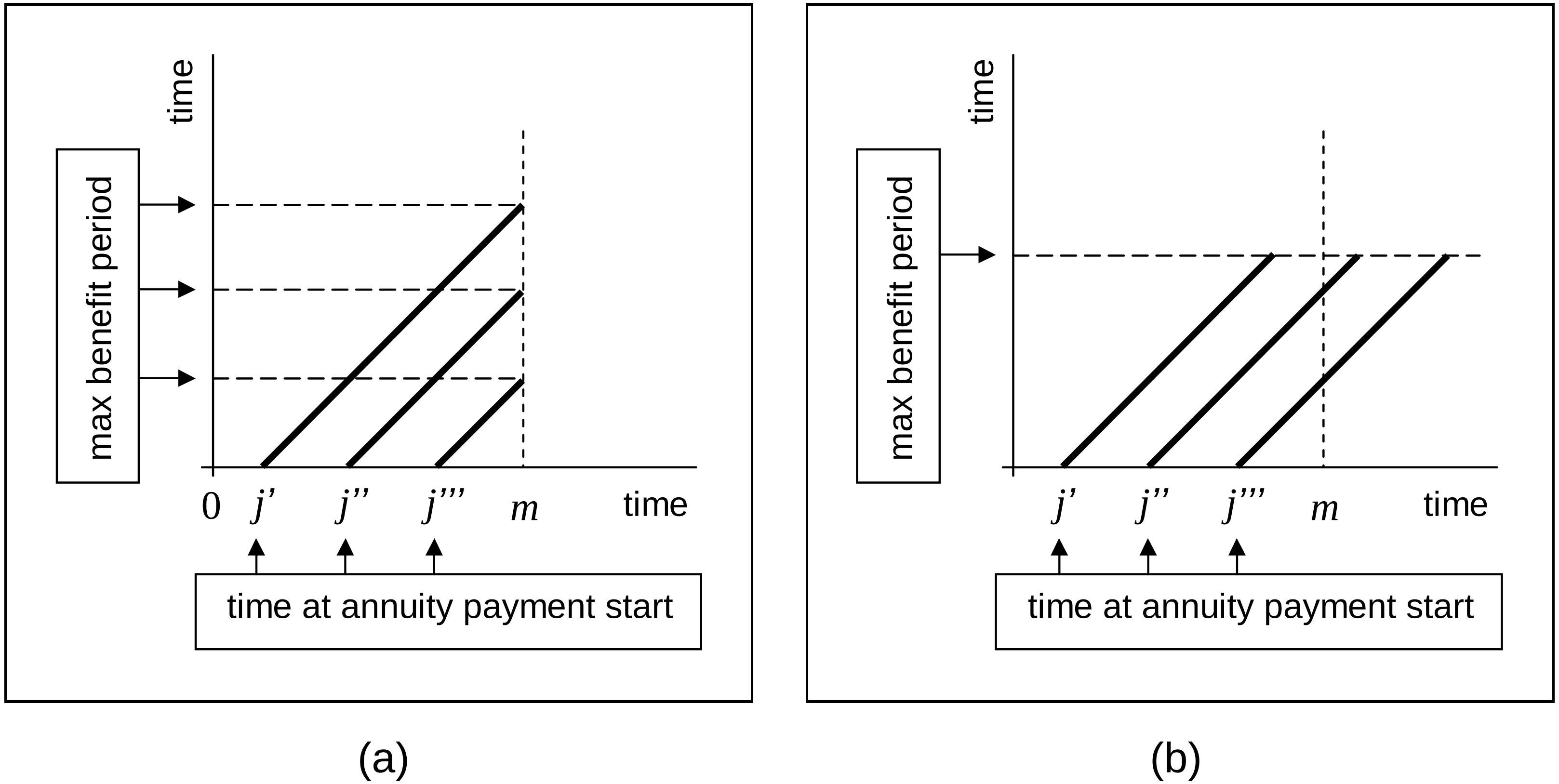
$$a^{ai}_{x:m\rceil} = \sum_{j=1}^{m} {}_{j-1}p^{aa}_{x}\; p^{ai}_{x+j-1}\; v^{j}\; \ddot{a}^{i}_{x+j:m-j+1\rceil}$$

$\Rightarrow$ *inception-annuity* formula

As regards the maximum benefit period:

- ▷ preceding formulae based on the assumption that the disability annuity is payable up to the policy term m $\Rightarrow$ stopping time coincides with the policy term, while maximum benefit period depends on the time at which the disability annuity starts
- ▷ policy conditions can state a maximum benefit period of s years, independent of the time at which the disability annuity starts; if s is large (compared to m) $\Rightarrow$ benefit payment may last well beyond the insured period

See following Figures



Maximum benefit period according to policy conditions

With maximum benefit period of s years:

$$a^{ai}_{x:m;s\rceil} = \sum_{j=1}^{m} {}_{j-1}p^{aa}_x \; p^{ai}_{x+j-1} \sum_{h=j}^{j+s-1} v^h \; {}_{h-j}p^{\underline{ii}}_{x+j}$$

or:

$$a^{ai}_{x:m;s\rceil} = \sum_{j=1}^{m} {}_{j-1}p^{aa}_x \; p^{ai}_{x+j-1} \; v^j \; \ddot{a}^{i}_{x+j:s\rceil}$$

A fixed maximum benefit period s is required if the insured period m is very short

Commonly $m = 1$ for each individual cover in a group insurance

If $m = 1$:

$$a^{ai}_{x:1;s\rceil} = p^{ai}_x \; v \; \ddot{a}^{i}_{x+1:s\rceil}$$

With reference to previous equations, quantities

$$p^{ai}_{x+j-1}\ v\ \ddot{a}^{i}_{x+j:\overline{m-j+1}|}$$
$$p^{ai}_{x+j-1}\ v\ \ddot{a}^{i}_{x+j:\overline{s}|}$$

represent, for $j = 1, 2, \ldots, m$, the annual expected related to an active insured age $x + j - 1 \ \Rightarrow$ *natural premiums* of the insurance covers

We can write:

$$p^{ai}_{x+j-1}\ v\ \ddot{a}^{i}_{x+j:\overline{m-j+1}|} = a^{ai}_{x+j-1:1;\overline{m-j+1}|}$$
$$p^{ai}_{x+j-1}\ v\ \ddot{a}^{i}_{x+j:\overline{s}|} = a^{ai}_{x+j-1:1;\overline{s}|}$$

and then:

$$a^{ai}_{x:m\rceil} = \sum_{j=1}^{m} {}_{j-1}p^{aa}_x \; v^{j-1} \; a^{ai}_{x+j-1:1;m-j+1\rceil}$$

$$a^{ai}_{x:m;s\rceil} = \sum_{j=1}^{m} {}_{j-1}p^{aa}_x \; v^{j-1} \; a^{ai}_{x+j:1;s\rceil}$$

$\Rightarrow$ actuarial values expressed as expected present values (related to an active insured age x) of the annual expected costs

Actuarial value of a temporary immediate annuity payable for m' years at most while the insured (assumed active at age x) is active:

$$\ddot{a}^{aa}_{x:m'\rceil} = \sum_{h=1}^{m'} v^{h-1} \, {}_{h-1}p^{aa}_x$$

$\Rightarrow$ used for calculating periodic level premiums (waived during the disability spells)

PREMIUMS

Focus on *net premiums* (i.e. only meeting the benefits and thus not accounting for insurer's expenses)

Premium calculation principle: *equivalence principle*
$\Rightarrow$ at policy issue:

actuarial value of premiums = actuarial value of benefits

Single premiums

Net single premium for annual benefit b disability benefit payable up to the policy term m:

$$\Pi_{x,m} = b\, a^{ai}_{x:m\rceil}$$

If a maximum benefit period of s years is stated:

$$\Pi_{x,m;s} = b\, a^{ai}_{x:m;s\rceil}$$

Annual level premiums

Premiums paid while the insured is active (and not while disabled $\Rightarrow$ premium waiver)

Assume premiums payable for m' years at most ($m' \leq m$)

- Annual benefit b payable up to the policy term m:

$$P_{x,m(m')}\, \ddot{a}^{aa}_{x:\overline{m'}|} = b\, a^{ai}_{x:\overline{m}|}$$

- Maximum benefit period of s years:

$$P_{x,m(m');s}\, \ddot{a}^{aa}_{x:\overline{m'}|} = b\, a^{ai}_{x:\overline{m;s}|}$$

Assume $m' = m \;\Rightarrow$ we find:

$$P_{x,m(m)} = b\,\frac{\sum_{j=1}^{m} {}_{j-1}p_x^{aa}\; v^{j-1}\; a^{ai}_{x+j-1:1;m-j+1\rceil}}{\sum_{j=1}^{m} v^{j-1}\,{}_{j-1}p_x^{aa}}$$

$$P_{x,m(m);s} = b\,\frac{\sum_{j=1}^{m} {}_{j-1}p_x^{aa}\; v^{j-1}\; a^{ai}_{x+j-1:1;s\rceil}}{\sum_{j=1}^{m} v^{j-1}\,{}_{j-1}p_x^{aa}}$$

In both cases: annual level premium = arithmetic weighted average of the natural premiums

If natural premiums decrease as the duration of the policy increases, then:

- ▷ level premium initially lower than the natural premiums
- ▷ insufficient funding of the insurer (negative reserve)
- ▷ shortened premiums ($m' < m$) needed

Example

Refer to an insurance product providing a disability annuity in the case of permanent or temporary disability

Annuity payable at policy anniversaries, while the insured is disabled, up to maturity m at most

Let $b = 100$ and $v = 1.02^{-1}$

Assume:

$$p_y^{ai} = 0.00223 \times 1.0468^y$$

For example:

$$p_{30}^{ai} = 0.008795, \;\; p_{45}^{ai} = 0.017465, \;\; p_{55}^{ai} = 0.027594, \;\; p_{60}^{ai} = 0.034684$$

Assume:

$$p_y^{ia} = \begin{cases} 0.05 & \text{for } \; y \leq 60 \\ 0 & \text{for } \; y > 60 \end{cases}$$

Let q_y = probability of dying between exact age y and $y+1$, irrespective of the state, assume given by Heligman-Pollard law (see Examples in: "Actuarial models for sickness insurance")

State-specific mortality:

$$q_y^a = q_y$$
$$q_y^i = (1+\gamma)\, q_y$$

with $\gamma = 0.25$

Finally:

$$p_y^{aa} = 1 - p_y^{ai} - q_y^a$$
$$p_y^{ii} = 1 - p_y^{ia} - q_y^i$$

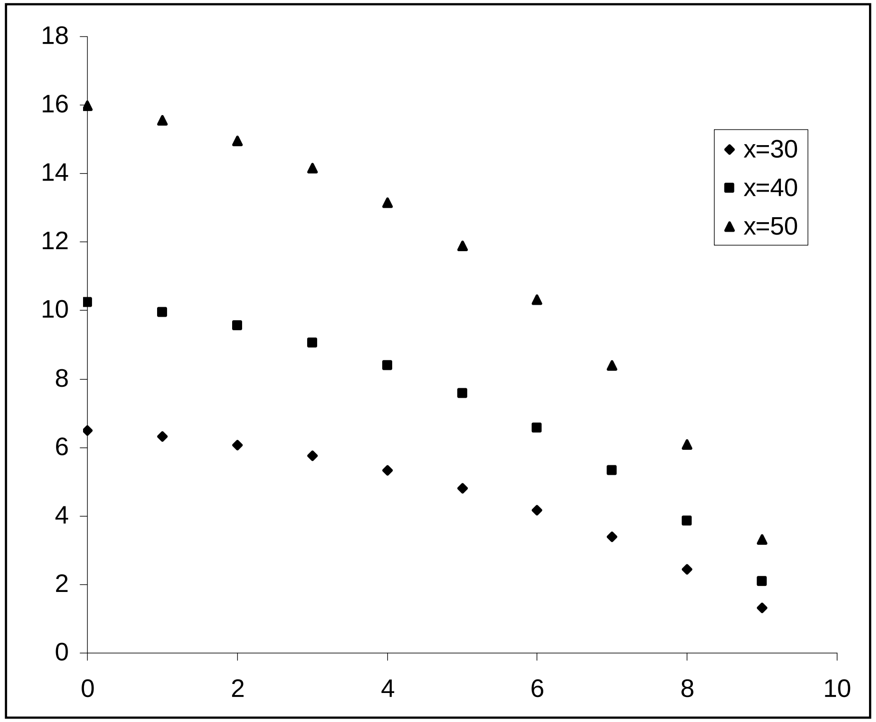
See following Tables and Figures

	$x = 30$	$x = 40$	$x = 50$
$m = 10$	41.656	64.219	96.918
	4.756	7.541	11.945
$m = 15$	84.185	127.360	190.563
	6.919	10.965	17.758
$m = 20$	136.777	202.044	311.067
	9.129	14.411	24.816

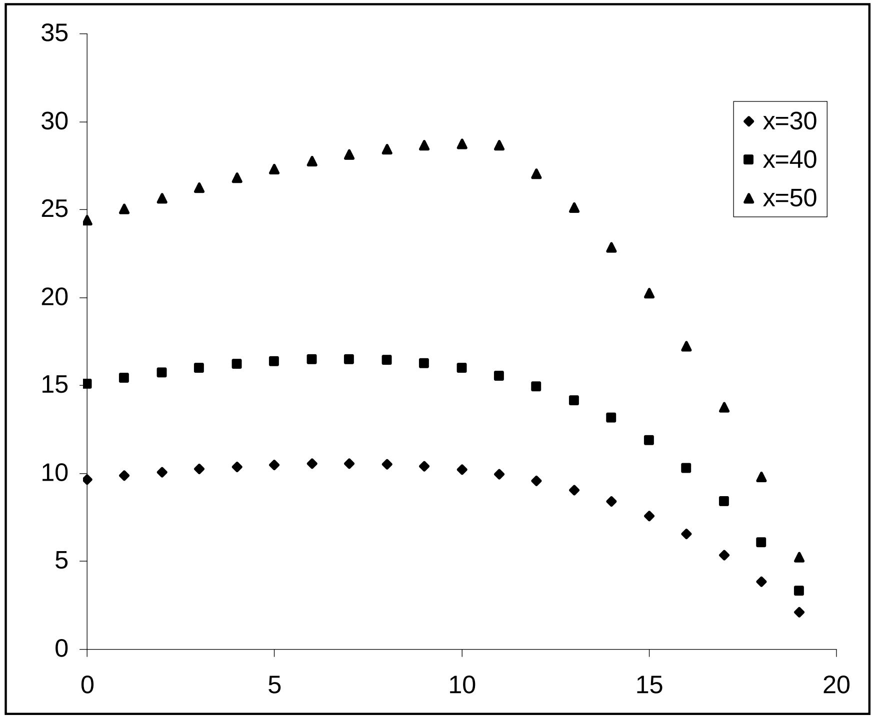
Single premiums and annual level premiums ($m' = m$)

	$x = 30$	$x = 40$	$x = 50$
$m = 10$, $m' = 7$	6.495	10.197	15.876
$m = 15$, $m' = 10$	9.611	14.956	23.487
$m = 20$, $m' = 15$	11.242	17.395	28.988

Annual level premiums



Natural premiums; $m = 10$



Natural premiums; $m = 20$

RESERVES

Policy (prospective) reserve, at (integer) time t:

actuarial value of future benefits - actuarial value of future premiums

given the state of the policy at time t

In disability insurance:

- ▷ *active reserve* (state a)
- ▷ *disabled reserve* (state i)

Active reserve

Refer to disability cover providing disability annuity up to policy term m at most

Level premiums payable for m' years

For simplicity, let $P = P_{x,m(m')}$

Active reserve at (integer) time t:

$$V_t^{(a)} = \begin{cases} b\, a^{ai}_{x+t:m-t\rceil} - P\, \ddot{a}^{aa}_{x+t:m'-t\rceil} & 0 \le t < m' \\ b\, a^{ai}_{x+t:m-t\rceil} & m' \le t \le m \end{cases}$$

with $V_0^{(a)} = V_m^{(a)} = 0$

Disabled reserve

Disabled reserve given by:

$$V_t^{(i)} = \begin{cases} b\,\ddot{a}^{ii}_{x+t:\overline{m-t}|} - P\,\ddot{a}^{ia}_{x+t:\overline{m'-t}|} & 0 \le t < m' \\ b\,\ddot{a}^{ii}_{x+t:\overline{m-t}|} & m' \le t \le m \end{cases}$$

Note:

- ▷ term $b\,\ddot{a}^{ii}_{x+t:\overline{m-t}|}$ = actuarial value of the running disability annuity as well as of possible future disability annuities after recovery
- ▷ term $P\,\ddot{a}^{ia}_{x+t:\overline{m'-t}|}$ = actuarial value of future premiums paid after possible recovery

Disregarding benefits and premiums after possible recovery

$\Rightarrow$ approx formula:

$$V_t^{(i)} = b\,\ddot{a}^{i}_{x+t:\overline{m-t}|}$$

Recursive relations

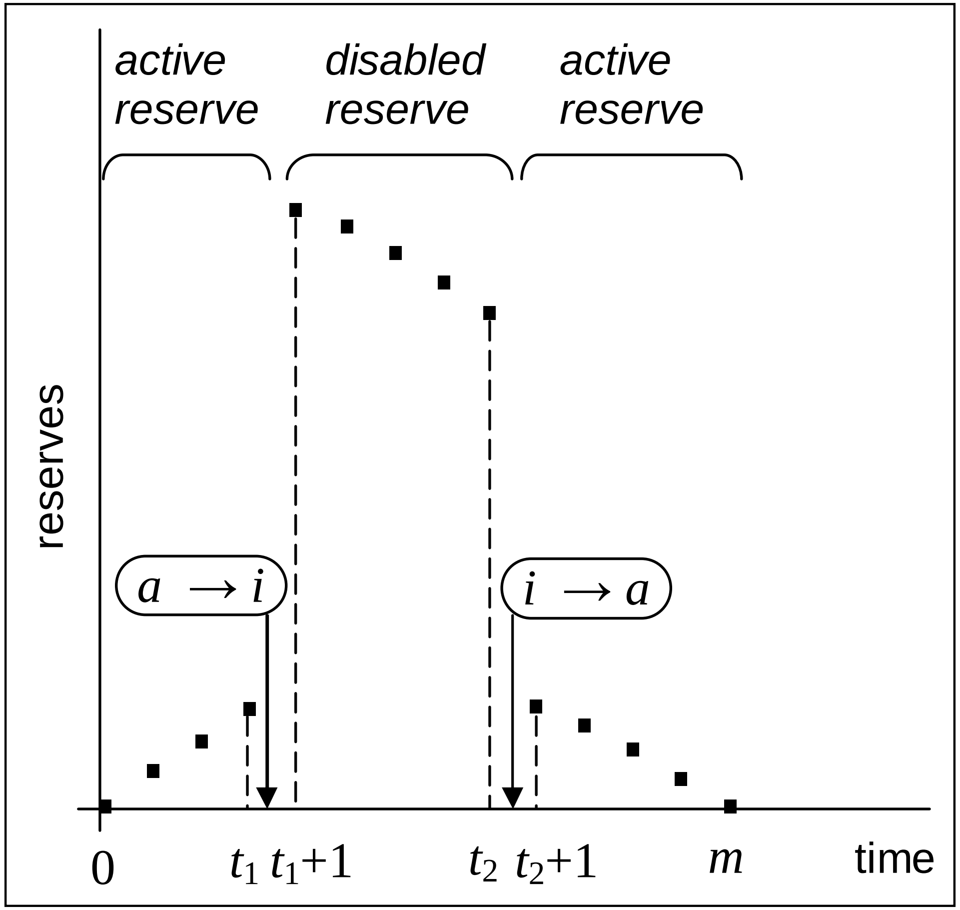
After several manipulations, recursive relations for the active reserve and the disabled reserve:

$$V_t^{(a)} + P = v\, V_{t+1}^{(a)} + v\, p_{x+t}^{ai} \left(V_{t+1}^{(i)} - V_{t+1}^{(a)} \right) - v\, q_{x+t}^{a}\, V_{t+1}^{(a)}$$

$$V_t^{(i)} - b = v\, V_{t+1}^{(i)} + v\, p_{x+t}^{ia} \left(V_{t+1}^{(a)} - V_{t+1}^{(i)} \right) - v\, q_{x+t}^{i}\, V_{t+1}^{(i)}$$

Interpretation $\Rightarrow$ as $V_{t+1}^{(i)} > V_{t+1}^{(a)}$ (see examples) we note that:

- $V_{t+1}^{(i)} - V_{t+1}^{(a)}$ = increase in the reserve profile because of $a \rightarrow i$
  $\Rightarrow$ financed via mutuality
- $V_{t+1}^{(a)} - V_{t+1}^{(i)}$ = decrease in the reserve profile because of $i \rightarrow a$
  $\Rightarrow$ amount released, shared in mutuality



Time-profile of the policy reserve: an example

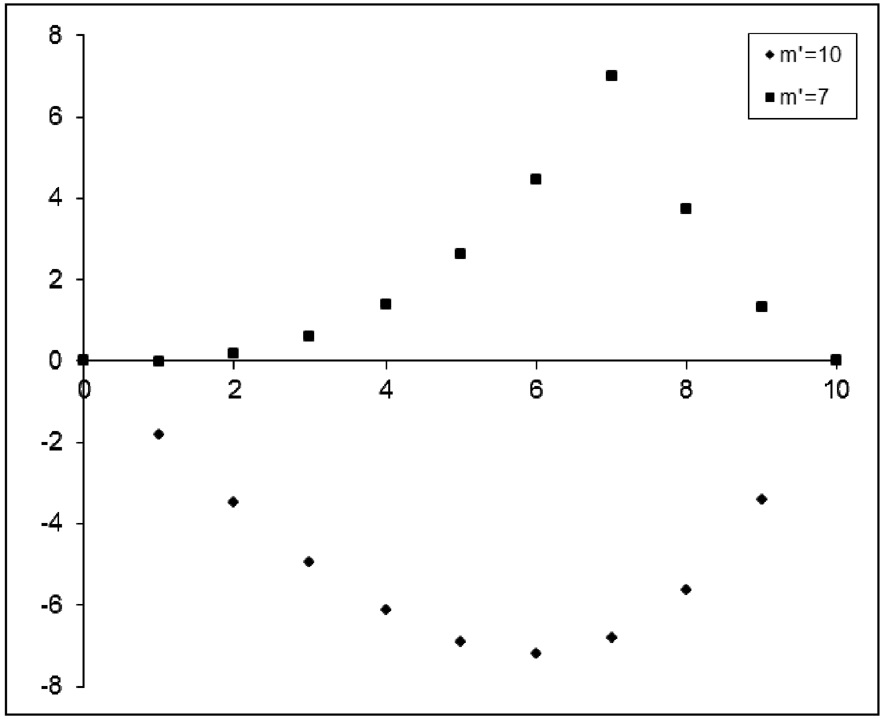
Example

Assume the technical basis adopted in previous Example

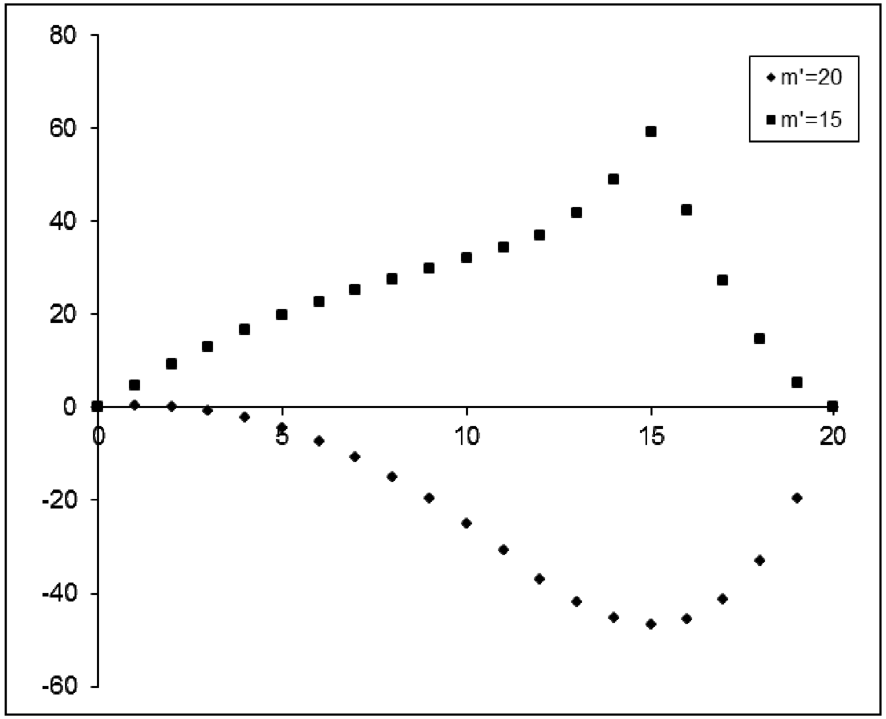
Following Figures:

- active reserve for policies with $m = 10$ and $m = 20$
  - ▷ premium payment must be shortened in order to avoid negative reserves (insurer's credit)
- disabled reserve (i.e. referred to an annuity in payment)
  - ▷ amount much higher than the active reserve

Actuarial models for disability annuities *(cont'd)*

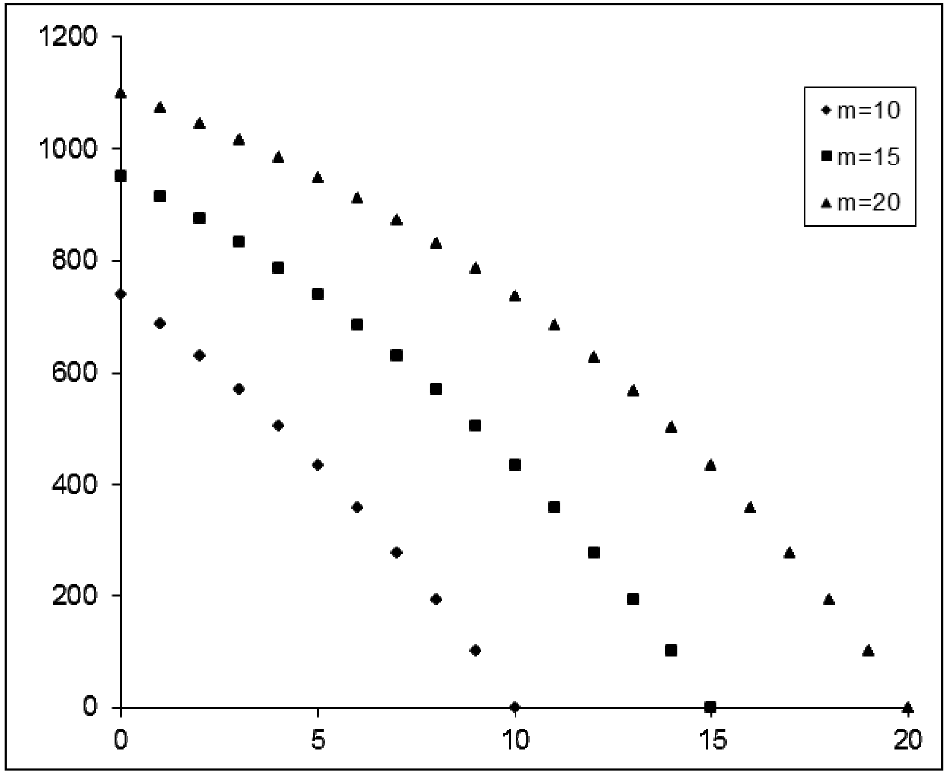


Active reserves; $x = 30, m = 10$



Active reserves; $x = 50, m = 20$

Actuarial models for disability annuities *(cont'd)*



Disabled reserves; $x = 30$

Reserves at fractional durations

Active reserve $\Rightarrow$ see formulae for aging reserve in sickness insurance

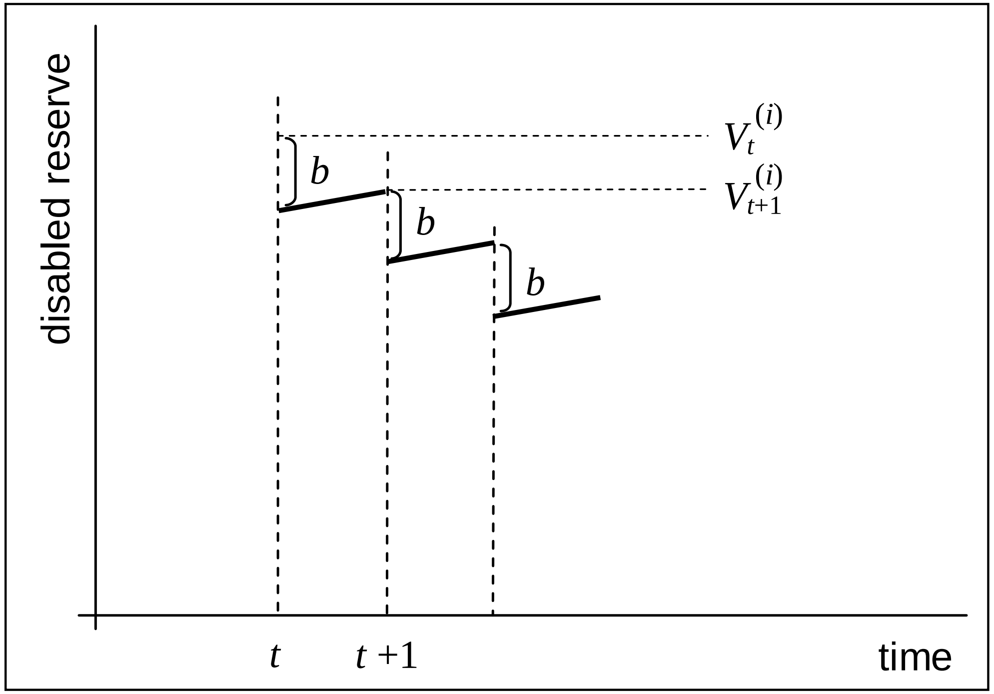
Disabled reserve:

$$V_{t+r}^{(i)} = (1-r)\,(V_t^{(i)} - b) + r\,V_{t+1}^{(i)}$$

for $t = 0, 1, \dots$ and $0 < r < 1$

See following Figure

Actuarial models for disability annuities *(cont'd)*



Interpolated profile of the disabled reserve

REPRESENTING POLICY CONDITIONS

Important policy conditions can be formally described by a set of five parameters:

$$\Gamma = [m_1, m_2, f, s, r]$$

where:

(m_1, m_2) = insured period; for example:

$m_1 = c$ = waiting period (from policy issue)

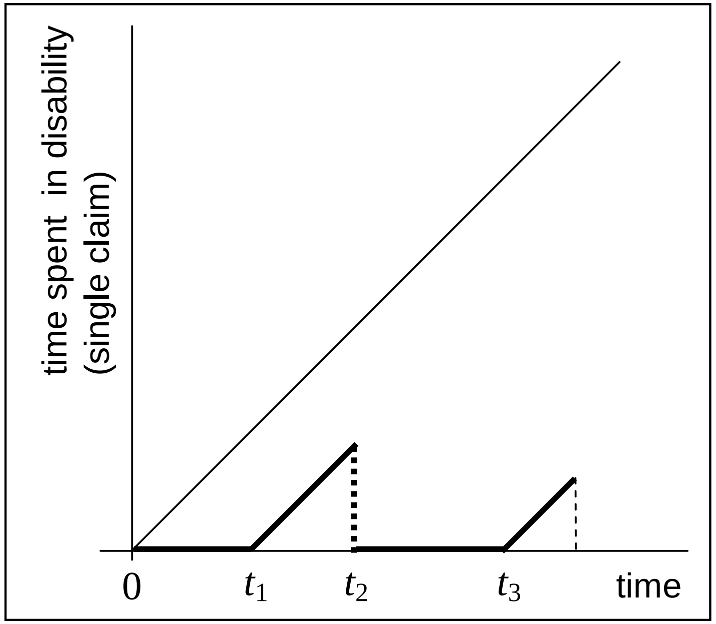
$m_2 = m$ = policy term

f = deferred period (from disability inception)

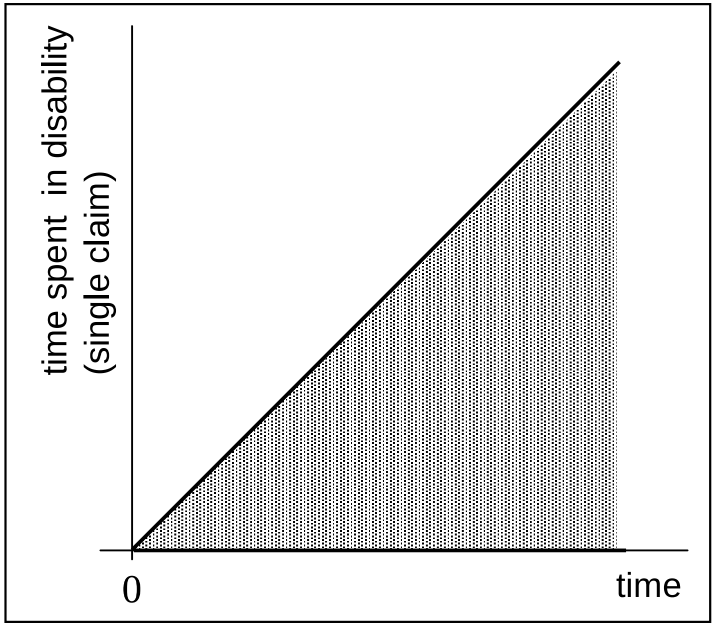
s = maximum benefit period, i.e. maximum number of years of annuity payment (from disability inception)

r = stopping time (from policy issue); for example, if x denotes the age at policy issue and ξ the retirement age, then we can set $r = \xi - x$

Assume that each individual "story" is represented according to the Lexis diagram logic

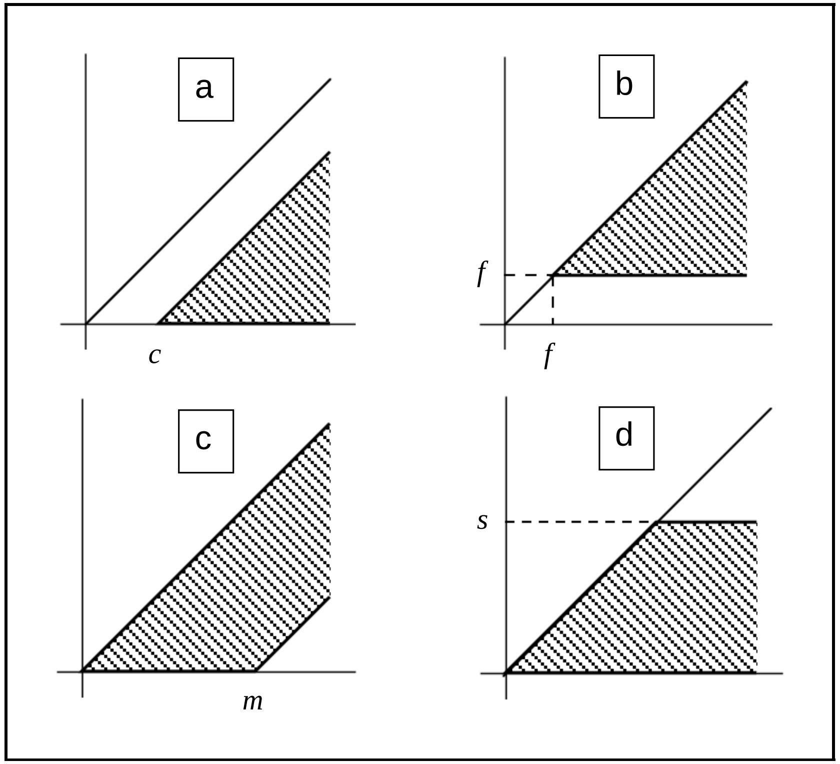


An example of disability story



The region of possible disability stories

Single premium $\Rightarrow$ a "measure" associated to a subset of the shaded region $\Rightarrow$ See following Figures



Policy conditions

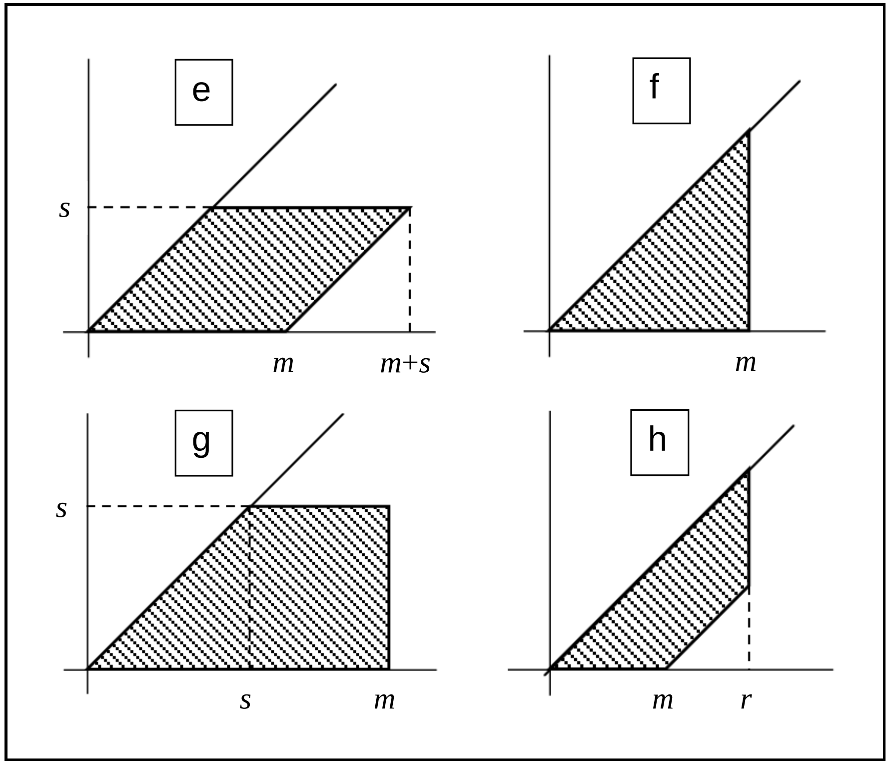
(a) $\Gamma = [c, \infty, 0, \infty, \infty]$

(b) $\Gamma = [0, \infty, f, \infty, \infty]$

(c) $\Gamma = [0, m, 0, \infty, \infty]$

(d) $\Gamma = [0, \infty, 0, s, \infty]$

Actuarial models for disability annuities *(cont'd)*



Policy conditions

(e) $\Gamma = [0, m, 0, s, \infty]$

(f) $\Gamma = [0, m, 0, \infty, m]$

(g) $\Gamma = [0, m, 0, s, m]$

(h) $\Gamma = [0, m, 0, \infty, r]$

Example 1

Annuity benefit payable up to policy term m; no waiting period, no deferred period

Conditions:

$$\Gamma = [0, m, 0, \infty, m]$$

Single premium:

$$\mathit{\Pi} = a^{ai}_{x:n\rceil} = \underbrace{\sum_{j=1}^{m}}_{\Uparrow} \underbrace{{}_{j-1}p^{aa}_{x}\, p^{ai}_{x+j-1} \Big[\underbrace{\sum_{h=j}^{m}}_{\Uparrow} \underbrace{v^{h}\; {}_{h-j}p^{\underline{ii}}_{x+j}}_{\text{terms 1st sum}} \Big]}_{\text{terms 2nd sum}}$$

$\mathit{\Pi}$ = double sum of values over the "region" defined by policy conditions

Example 2

Annuity benefit payable for s years max; no waiting period, no deferred period

Policy conditions:

$$\Gamma = [0, m, 0, s, \infty]$$

Single premium:

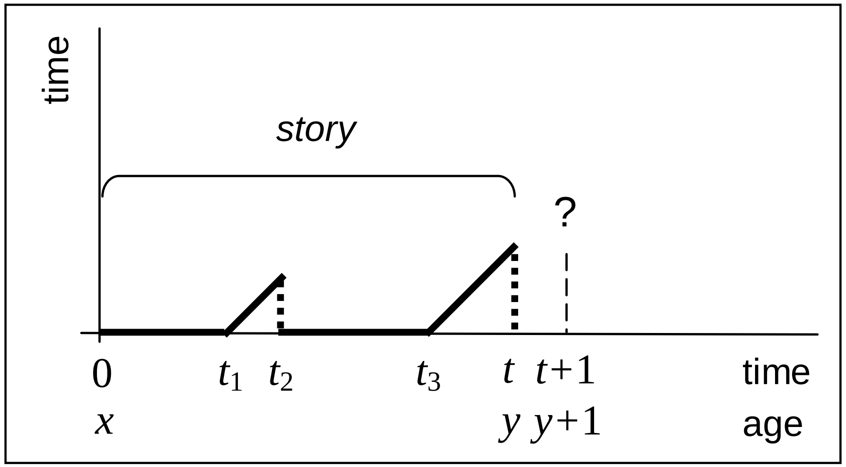
$$\Pi = a^{ai}_{x:n\rceil} = \underbrace{\sum_{j=1}^{m}}_{\Uparrow} \underbrace{{}_{j-1}p^{aa}_{x}\, p^{ai}_{x+j-1} \Big[\underbrace{\sum_{h=j}^{j+s-1}}_{\Uparrow} \underbrace{v^{h}\,{}_{h-j}p^{\underline{ii}}_{x+j}}_{\text{terms 1st sum}} \Big]}_{\text{terms 2nd sum}}$$

ALLOWING FOR DURATION EFFECTS

Probabilistic model defined above: transition probabilities at any age y only depend on the current state at that age

Refer to following Figure:

- ▷ probabilities (assessed at time t) concerning state in $t+1$
- ▷ available information: individual story from 0 to t



Story of an insured risk

For example, as regards transition $i \rightarrow a$ between age y and $y+1$, the following probabilities can in principle be considered:

(1) $\mathbb{P}[\text{in } a \text{ at age } y+1 \mid \text{time elapsed since policy issue}]$

(2) $\mathbb{P}[\text{in } a \text{ at age } y+1 \mid \text{story up to age } y]$

(3) $\mathbb{P}[\text{in } a \text{ at age } y+1 \mid \text{total time in } i \text{ up to age } y]$

(4) $\mathbb{P}[\text{in } a \text{ at age } y+1 \mid \text{time in } i \text{ since the latest transition into } i]$

(5) $\mathbb{P}[\text{in } a \text{ at age } y+1 \mid \text{number of transitions into } i \text{ up to age } y]$

(6) $\mathbb{P}[\text{in } a \text{ at age } y+1 \mid \text{in } i \text{ at age } y]$

In the Figure:

(1) time $= t$ (i.e. the past duration of the policy)

(2) story $= (a, i, a, i;\, t_1, t_2, t_3)$

(3) total time $= (t_2 - t_1) + (t - t_3)$

(4) time $= (t - t_3)$ (i.e. the past duration of the current disability spell)

(5) number $= 2$

Probabilistic and computational features

(1) $\Rightarrow$ *duration-since-issue dependence*, implies the use of *issue-select* probabilities, i.e. functions of both x and t (rather than functions the attained age $y = x + t$ only); for example, issue selection in the probability of $a \rightarrow i$ can represent a lower risk of disablement because of sickness thanks to medical ascertainment at policy issue

(2) $\Rightarrow$ serious difficulties in finding appropriate models to link transition probability to any possible past story

(6) $\Rightarrow$ *Markov model*, so far adopted; see for example probabilities

$$p_y^{aa},\ p_y^{ia}, \dots,\ q_y^{aa},\ q_y^{i}, \dots$$

simple implementation, widely adopted in the actuarial practice

(3), (4), (5) $\Rightarrow$ complex non-Markov models; possible shift to Markov models via approximations

In particular, dependence (4): *duration-in-current-state dependence* requires *inception-select* probabilities depending on both the attained age $y = x + t$ and the time z spent in the current state ("inception"= time at which the latest transition to that state occurred)

Practical issues $\Rightarrow$ focus on transitions from state i, i.e. disability duration effect on recovery and mortality of disabled lives

Statistical evidence $\Rightarrow$ initial "acute" phase and then a "chronic" phase

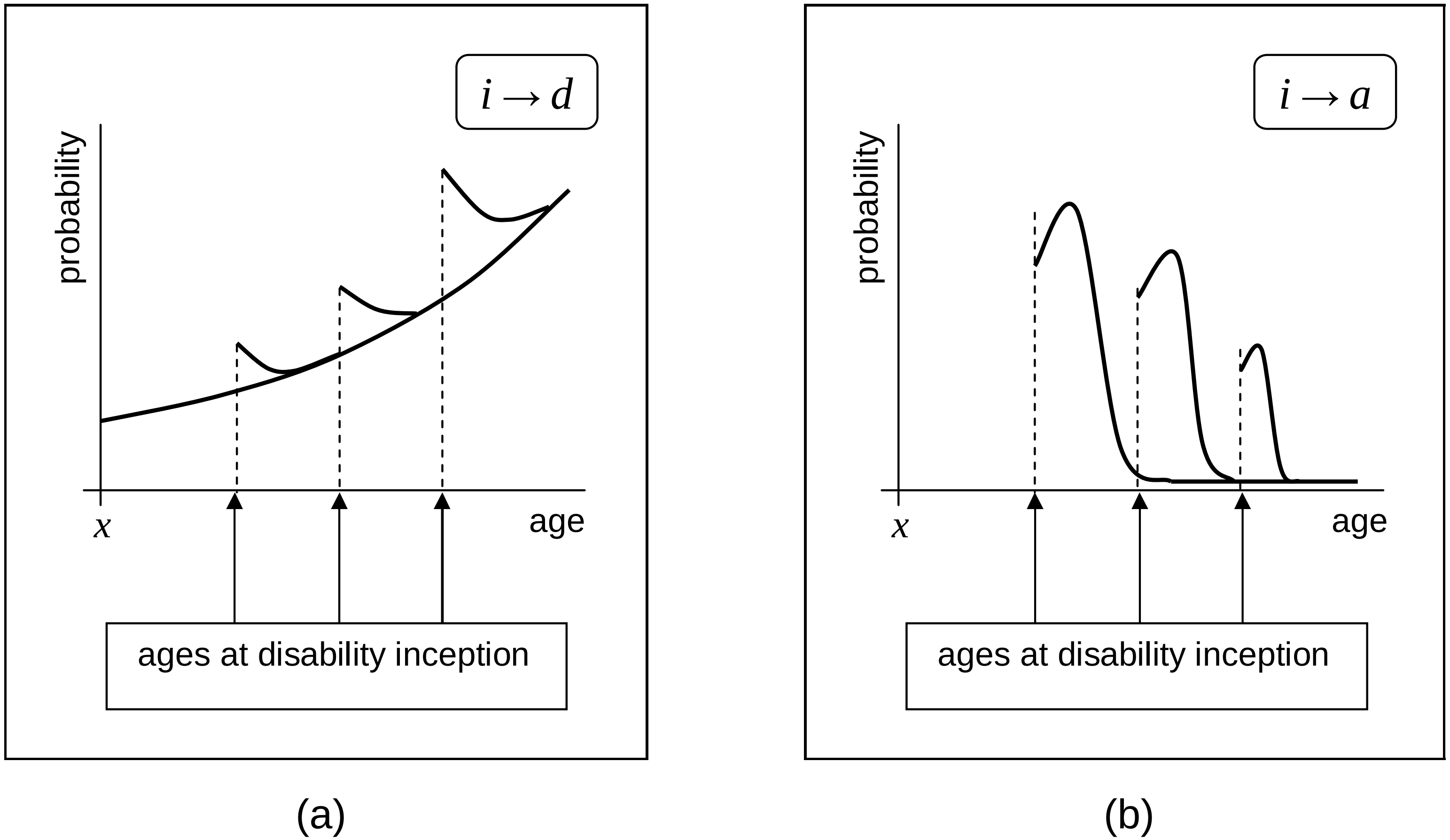
See following Figures

Some examples of inception-select probabilities (model (4)):

$$p^{ia}_{[y]} = \mathbb{P}[\text{in } a \text{ at age } y+1 | \text{ disability inception at age } y] \quad \text{(i.e. } z = 0\text{)}$$

$$p^{ia}_{[y-z]+z} = \mathbb{P}[\text{in } a \text{ at age } y+1 | \text{ disability inception at age } y-z]$$

$$_k p^{ii}_{[y]} = \mathbb{P}[\text{in } i \text{ up to age } y+k | \text{ disability inception at age } y] \quad \text{(i.e. } z = 0\text{)}$$



Effect of time spent in the disability state

Implementing dependence (4) working with a Markov model
$\Rightarrow$ splitting the disability state i into n states, $i^{(1)}, i^{(2)}, \ldots, i^{(n)}$, which represent disability according to duration since disablement

For example:

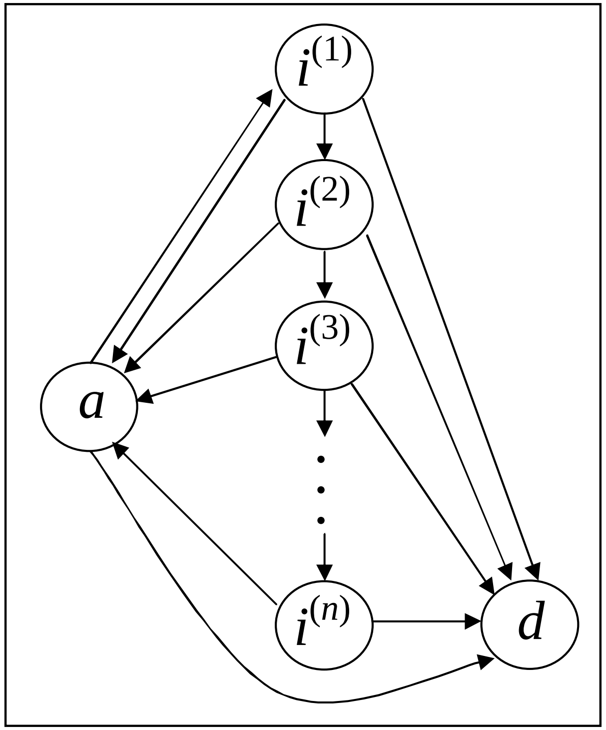
> $i^{(h)}$ = the insured is disabled with a duration of disability between $h-1$ and h, for $h = 1, 2, \ldots, n-1$
>
> $i^{(n)}$ = the insured is disabled with a duration of disability greater than $n-1$

$\Rightarrow$ resulting model called the "Dutch model"

See following Figure and transition matrix

Reasonable assumptions $\Rightarrow$ for example, for any age y:

$$p_y^{i^{(1)}a} > p_y^{i^{(2)}a} > \cdots > p_y^{i^{(n)}a} \geq 0$$



The "Dutch model"

	state at age $y+1$					
state at age y	a	$i^{(1)}$	$i^{(2)}$	$\dots$	$i^{(n)}$	d
a	p_y^{aa}	$p_y^{ai^{(1)}}$	0	$\dots$	0	q_y^{a}
$i^{(1)}$	$p_y^{i^{(1)}a}$	0	$p_y^{i^{(1)}i^{(2)}}$	$\dots$	0	$q_y^{i^{(1)}}$
$i^{(2)}$	$p_y^{i^{(2)}a}$	0	0	$\dots$	0	$q_y^{i^{(2)}}$
$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$	$\dots$
$i^{(n)}$	$p_y^{i^{(n)}a}$	0	0	$\dots$	$p_y^{i^{(n)}i^{(n)}}$	$q_y^{i^{(n)}}$
d	0	0	0	$\dots$	0	1

Conditional probabilities of being in states a, $i^{(1)}$, $i^{(2)}$, $\dots$, $i^{(n)}$, d, at age $y+1$

Simplified implementation of dependence (5) (probabilities depending on the number of disability events) via Markov model

Define the following states:

a_0 = active, no previous disability

i_0 = disabled, no previous disability

a_1 = active, previously disabled

i_1 = disabled, previously disabled

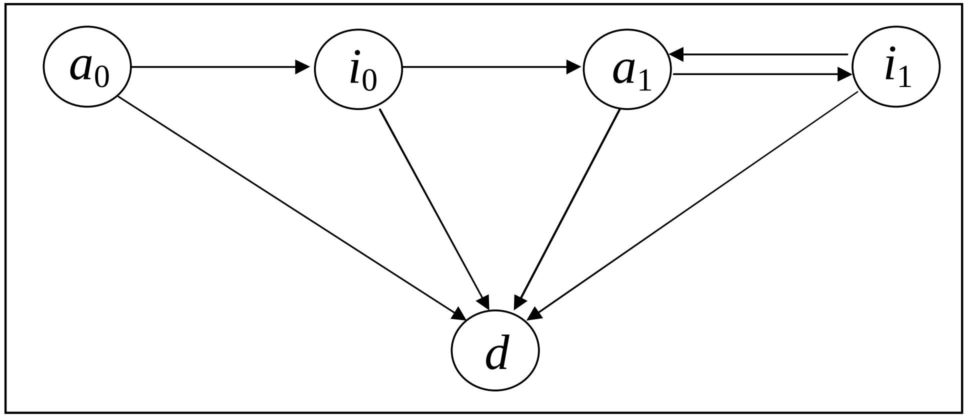
See following Figure and transition matrix

Underlying assumptions: for an insured with previous disability spells:

- higher probability of disablement
- higher probability of dying
- lower probability of recovery

For example, for any age y:

$$p_y^{i_0 a_1} > p_y^{i_1 a_1}; \quad p_y^{a_1 i_1} > p_y^{a_0 i_0}$$



A model with active and disabled states split according to previous disability

	state at age $y+1$				
state at age y	a_0	i_0	a_1	i_1	d
a_0	$p_y^{a_0 a_0}$	$p_y^{a_0 i_0}$	0	0	$q_y^{a_0}$
i_0	0	$p_y^{i_0 i_0}$	$p_y^{i_0 a_1}$	0	$q_y^{i_0}$
a_1	0	0	$p_y^{a_1 a_1}$	$p_y^{a_1 i_1}$	$q_y^{a_1}$
i_1	0	0	$p_y^{i_1 a_1}$	$p_y^{i_1 i_1}$	$q_y^{i_1}$
d	0	0	0	0	1

Conditional probabilities of being is states a_0, i_0, a_1, i_1, d, at age $y+1$

PRACTICAL ACTUARIAL APPROACHES

Actuarial models for Health Insurance products $\Rightarrow$ a mix of non-life insurance and life insurance features

Disability insurance (IP annuities) and LTCI:

- long-term contracts
- annuity-like benefits (lifelong in LTCI)

Hence, need for:

- ▷ biometric assumptions (in particular, lifetime probability distribution)
- ▷ financial aspects (investment, interest rate guarantee)

Biometric assumptions other than those required for life insurance and life annuities:

- probability of entering a disability state
- probability of leaving a disability state (mortality, recovery)

Further:

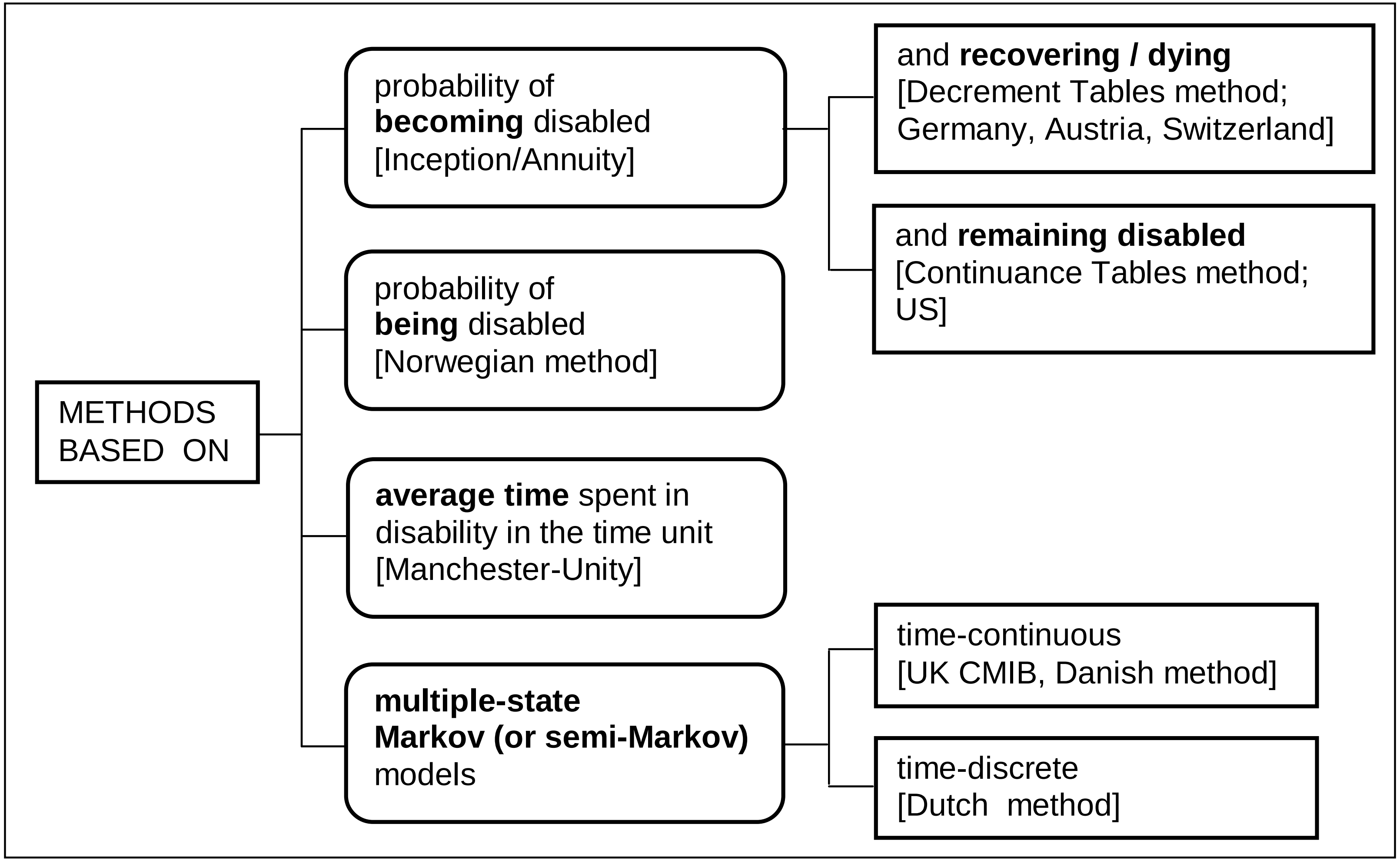
- statistical experience shows the impact of time spent in disability state on probabilities of leaving that state $\Rightarrow$ *inception-select* probabilities
- non-Markov models should be used to express the probabilistic structure

Data scarcity $\Rightarrow$ various approx calculation methods, in several cases disregarding the disability past-duration effect

Assume statistical data of a given type available according to a given format $\Rightarrow$ (approximate) calculation procedures often chosen consistently with type and format

Following Figure: a classification of actuarial methods for disability annuities (IP), including methods adopted in actuarial practice

Actuarial models for disability annuities *(cont'd)*



A classification of approaches to actuarial calculations for Income Protection

Converting data

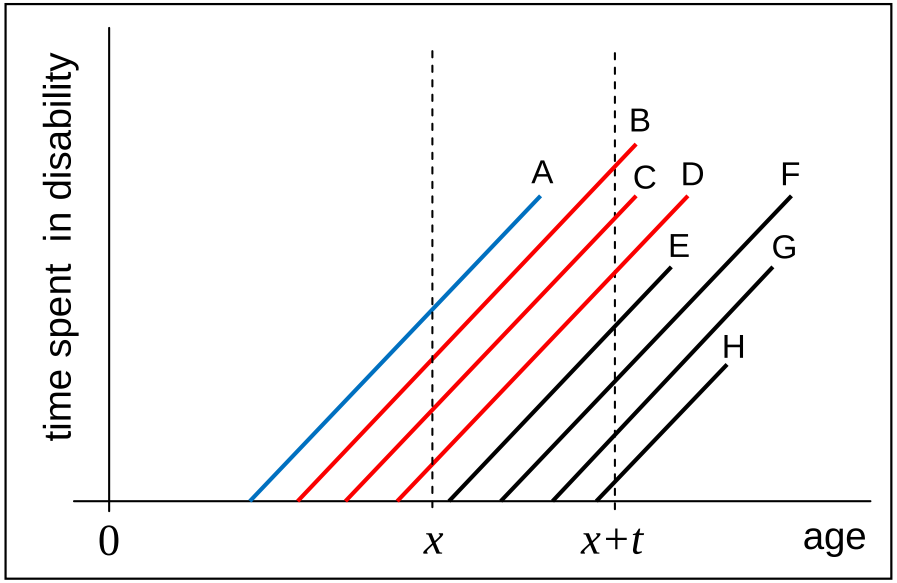
Assume that disability data are available as *prevalence rates*:

$$\frac{\text{number of people disabled at age } y}{\text{number of people alive at age } y}$$

Data available e.g. from social security database, or public health system database

These date cannot be directly used for insurance purposes, e.g. to assess the probability of *being disabled*, as they do not assume the individual was healthy at a given age, viz the age at policy issue

See following Figure



Some individual disability stories in a population

Refer to a portfolio consisting of a cohort entering insurance at age x

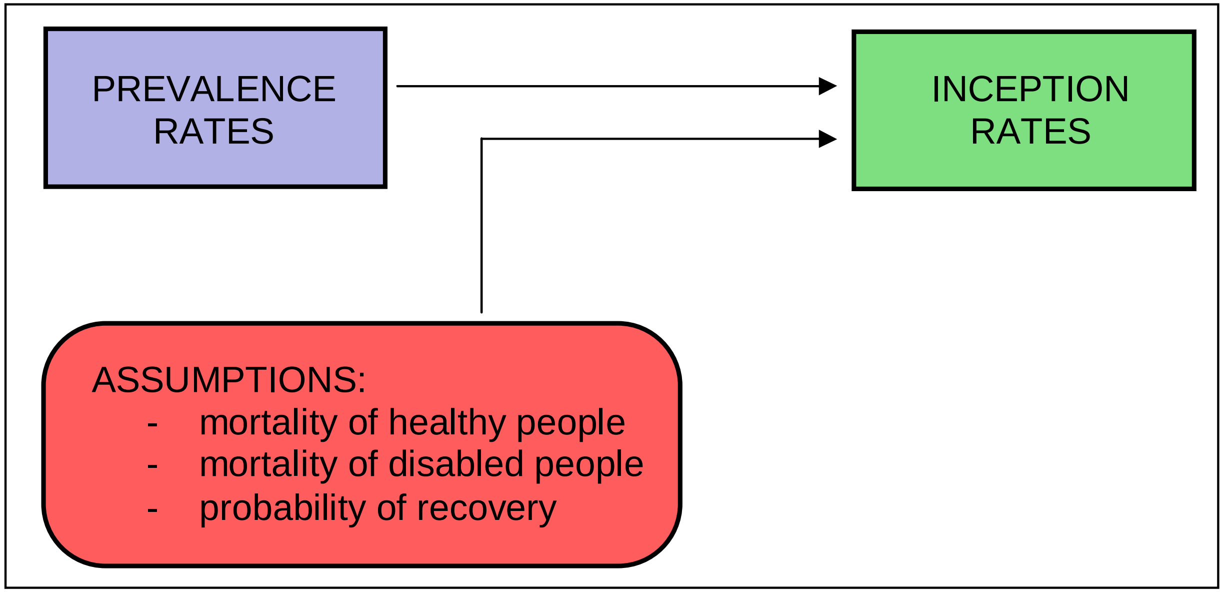
Individuals B, C and D (in the population), disabled at age $x+t$, should not be accounted for when determining the disability prevalence rate at age $x+t$, because entered the disability state before age x

Two basic approaches available

- Adjustment of the prevalence rates
  - ▷ j_{x+t} = prevalence rate at age $x + t$ (smoothed frequency)
  - ▷ define: $j_{(x)+t} = j_{x+t}\, \alpha(t)$ ($\alpha(t)$ = adjustment coefficient)
  - ▷ take $j_{(x)+t}$ as the probability of an individual healthy at age x being disabled at age $x + t$
  - ▷ method implemented in Norway
- Converting prevalence rates into *inception rates* $\Rightarrow$ probabilities of *becoming disabled*
  - ▷ set of (critical) assumptions needed

  See following Figure

Actuarial models for disability annuities *(cont'd)*



Converting disability data

ACTUARIAL MODELS FOR LTCI: AN INTRODUCTION

Focus on LTCI products which provide graded annuity benefits, i.e. benefits whose amount is graded according to the insured's disability degree

A basic biometric model

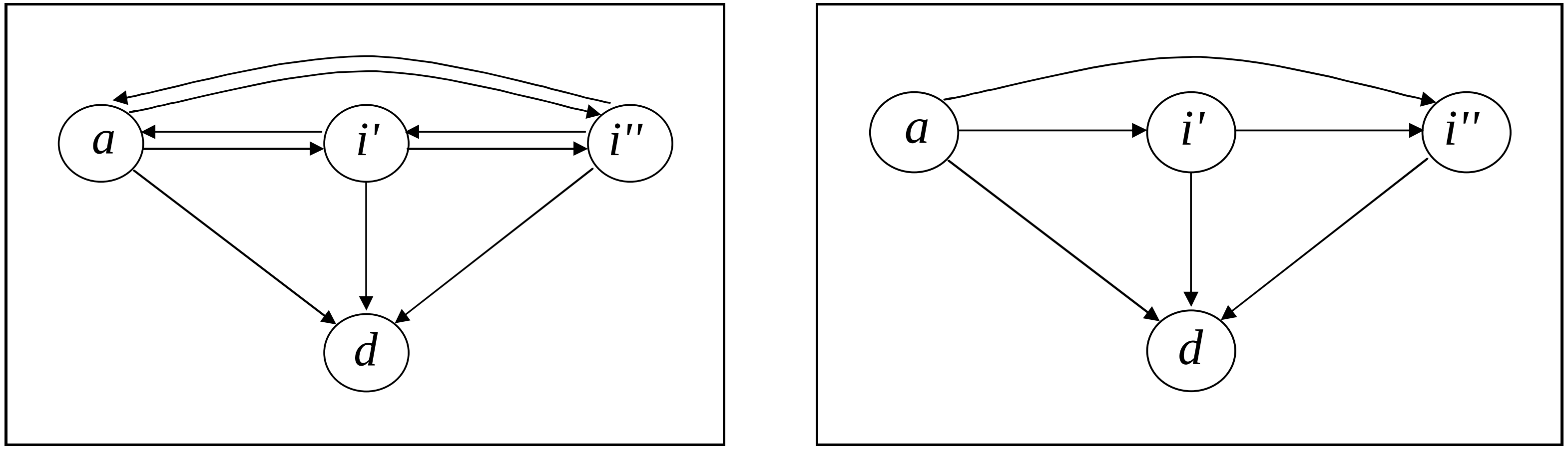
Disability degree expressed in terms of a (small) number of disability states $\Rightarrow$ actuarial model can be based on a multistate structure

See following Figures and transition matrix

Note that:

- ▷ more than one disability state to represent graded benefits (for example, see Figure (a) with two disability states)
- ▷ simplified structure can be adopted if the possibility of recovery is disregarded (see Figure (b))

Actuarial models for disability annuities *(cont'd)*



(a) (b)

Four-state models for LTC

Actuarial models for disability annuities *(cont'd)*

state at age y	state at age $y+1$			
	a	i'	i''	d
a	p_y^{aa}	$p_y^{ai'}$	$p_y^{ai''}$	q_y^{a}
i'	0	$p_y^{i'i'}$	$p_y^{i'i''}$	$q_y^{i'}$
i''	0	0	$p_y^{i''i''}$	$q_y^{i''}$
d	0	0	0	1

Conditional probabilities related to LTCI products

Following steps:

- from one-year transition probabilities to multi-year transition probabilities
- calculation of actuarial values $\Rightarrow$ premiums, reserves
  - ▷ reserve for healthy lives
  - ▷ reserves for disabled lives

Longevity risk issues

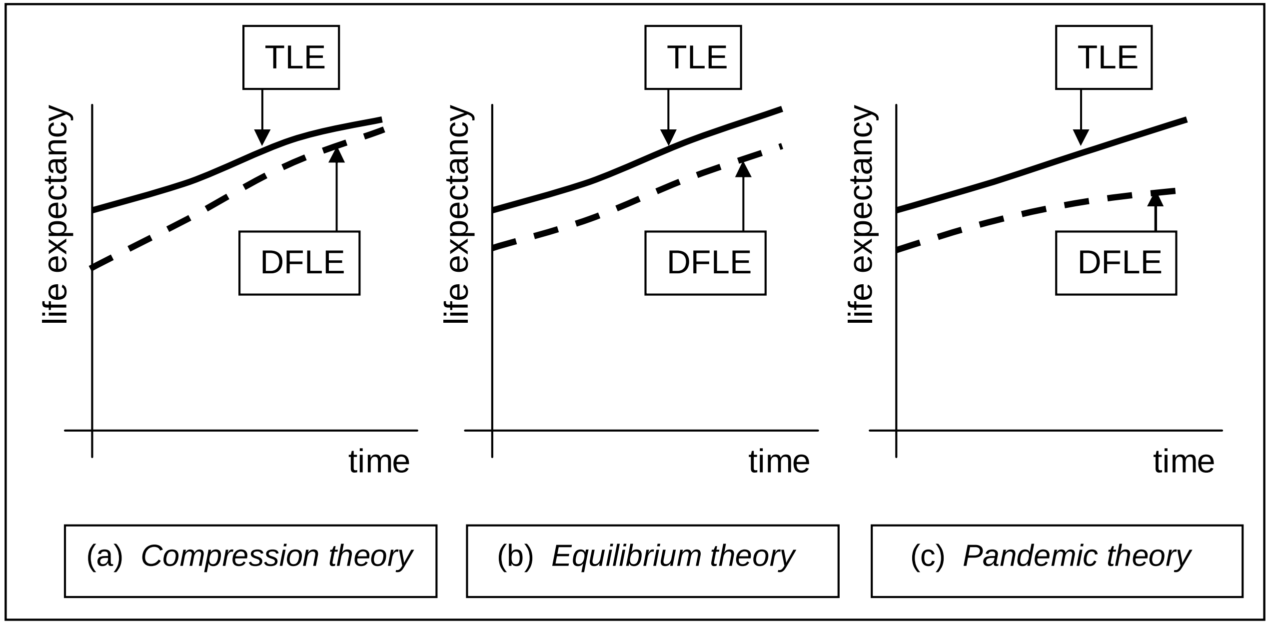
In all lifelong living benefits (i.e. life annuities, lifelong sickness insurance covers, LTCI annuities, etc.) the insurer bears the longevity risk, and in particular its systematic component, i.e. the *aggregate longevity risk* (possibility that all the insureds live, on average, longer than expected)

In the case of health insurance products, e.g. LTCI covers, risk emerges further from uncertainty concerning the time spent in the disability state

Three main theories proposed about the evolution of senescent disability (see following Figure)

Most important features of the three theories expressed in terms of the evolution of total life expectancy (TLE) and disability-free life expectancy (DFLE)

To assess the risks inherent in LTC covers ⇒ uncertainty in future mortality and disability trends should explicitly be taken into account ⇒ several scenarios must be considered, each one including a specific projection of mortality and disability trends



Trends in total life expectancy (TLE) and disability-free life expectancy (DFLE), according to different theories

LTCI PREMIUMS: SENSITIVITY ANALYSIS

INTRODUCTION

LTCI products are rather recent $\Rightarrow$ senescent disability data are scanty $\Rightarrow$ uncertainty in technical bases $\Rightarrow$ pricing difficulties

High premiums, in particular because of safety loading $\Rightarrow$ obstacle to the diffusion of these products (especially stand-alone LTC covers only providing “protection”)

Uncertainty in technical bases, in particular biometric assumptions:

- probability of disablement, i.e. entering LTC state
- probability of recovery. i.e. back to healthy state
- mortality of disabled people, i.e. lives in LTC state

Need for:

- ▷ accurate *sensitivity analysis*
- ▷ focus on *product design* $\Rightarrow$ single out products whose premiums (and reserves) are not too heavily affected by the choice of the biometric assumptions

Additional references:

B. D. Rickayzen. An analysis of disability-linked annuities. Faculty of Actuarial Science and Insurance, Cass Business School, City University, London. Actuarial Research Paper No. 180, 2007. Available at:
`http://www.cass.city.ac.uk/__data/assets/pdf_file/0018/37170/180ARP.pdf`

B. D. Rickayzen and D. E. P. Walsh. A multi-state model of disability for the United Kingdom: Implications for future need for Long-Term Care for the elderly. *British Actuarial Journal*, 8:341–393, 2002

LONG-TERM CARE INSURANCE (LTCI) PRODUCTS

The following products will be addressed in the sensitivity analysis

Stand-alone LTCI

(Product P1)

LTCI benefit: a lifelong annuity with predefined annual amount

LTCI as an acceleration benefit in a whole-life assurance

(Product P2(s))

Annual LTC benefit = $\dfrac{\text{sum assured}}{s}$ paid for s years at most

Package including LTC benefits and lifetime-related benefits

(Products P3a($x+n$) and P3b($x+n$))

Benefits:

(I) a lifelong LTC annuity (from the LTC claim on)

(II) a deferred life annuity from age $x+n$ (e.g. $x+n=80$), while the insured is not in LTC disability state

(III) a lump sum benefit on death, alternatively given by

- (IIIa) a fixed amount, stated in the policy
- (IIIb) the difference (if positive) between a fixed amount and the total amount paid as benefit 1 and/or benefit 2

Benefits (I) and (II) are mutually exclusive

Enhanced pension (Life care pension)

(Product P4(b', b''))

LTC annuity benefit defined as an uplift with respect to the basic pension b

Uplift financed by a reduction (with respect to the basic pension b) of the benefit paid while the policyholder is healthy

- ▷ reduced benefit b' paid as long as the retiree is healthy
- ▷ uplifted lifelong benefit b'' paid in the case of LTC claim

Of course, $b' < b < b''$

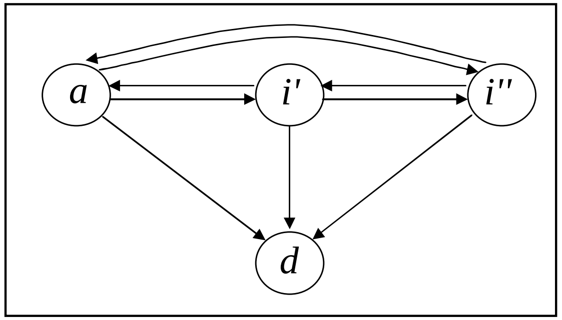
THE ACTUARIAL MODEL

Multistate models for LTCI

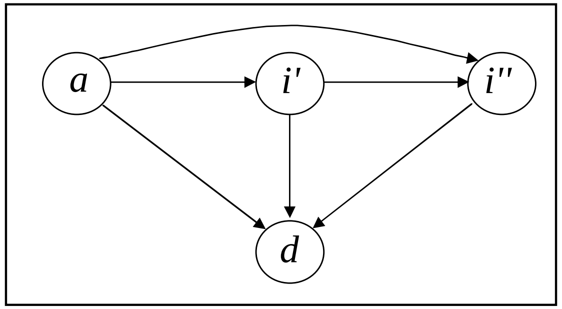
States:

- $a =$ active = healthy
- $i =$ invalid = in LTC state
- $d =$ died
- $i' =$ in low-severity LTC state
- $i'' =$ in high-severity LTC state

LTCI premiums: sensitivity analysis *(cont'd)*

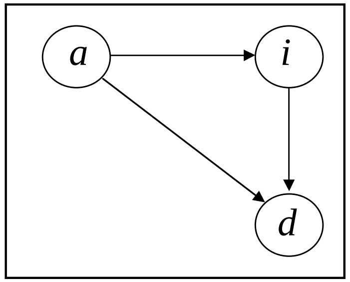


(a)

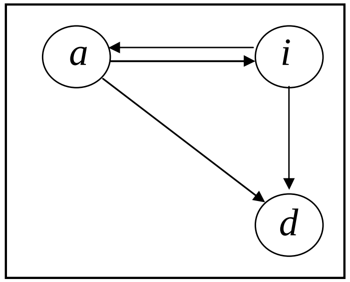


(b)

Four-state models



(a)



(b)

Three-state models

In what follows we adopt the three-state model (a), in a time-discrete context

Biometric functions (needed)

Refer to three-state model (a)

For an active (healthy) individual age x:

$q_x^{aa} =$ prob. of dying before age $x+1$ from state a

$w_x =$ prob. of becoming invalid (disablement, i.e. LTC claim) before $x+1$

For an invalid age x:

$q_x^i =$ prob. of dying before age $x+1$

Remark

No dependence on time elapsed since disability inception is allowed for
$\Rightarrow$ a Markov chain model is then adopted

TECHNICAL BASES

Assumptions

q_x^{aa}: life table (first Heligman-Pollard law)

w_x: a specific parametric law

$q_x^i = q_x^{aa} +$ extra-mortality (i.e. additive extra-mortality model)

Life table

First Heligman-Pollard law:

$$\frac{q_x^{aa}}{1 - q_x^{aa}} = a^{(x+b)^c} + d\, \mathrm{e}^{-e\,(\ln x - \ln f)^2} + g\, h^x$$

LTCI premiums: sensitivity analysis *(cont'd)*

a	b	c	d	e	f	g	h
0.00054	0.01700	0.10100	0.00014	10.72	18.67	2.00532×10^{-6}	1.13025

The first Heligman-Pollard law: parameters

$\mathring{e}_0$	$\mathring{e}_{40}$	$\mathring{e}_{65}$	Lexis	q_{80}^{aa}
85.128	46.133	22.350	90	0.03475

The first Heligman-Pollard law: some markers

Disablement (LTC claim)

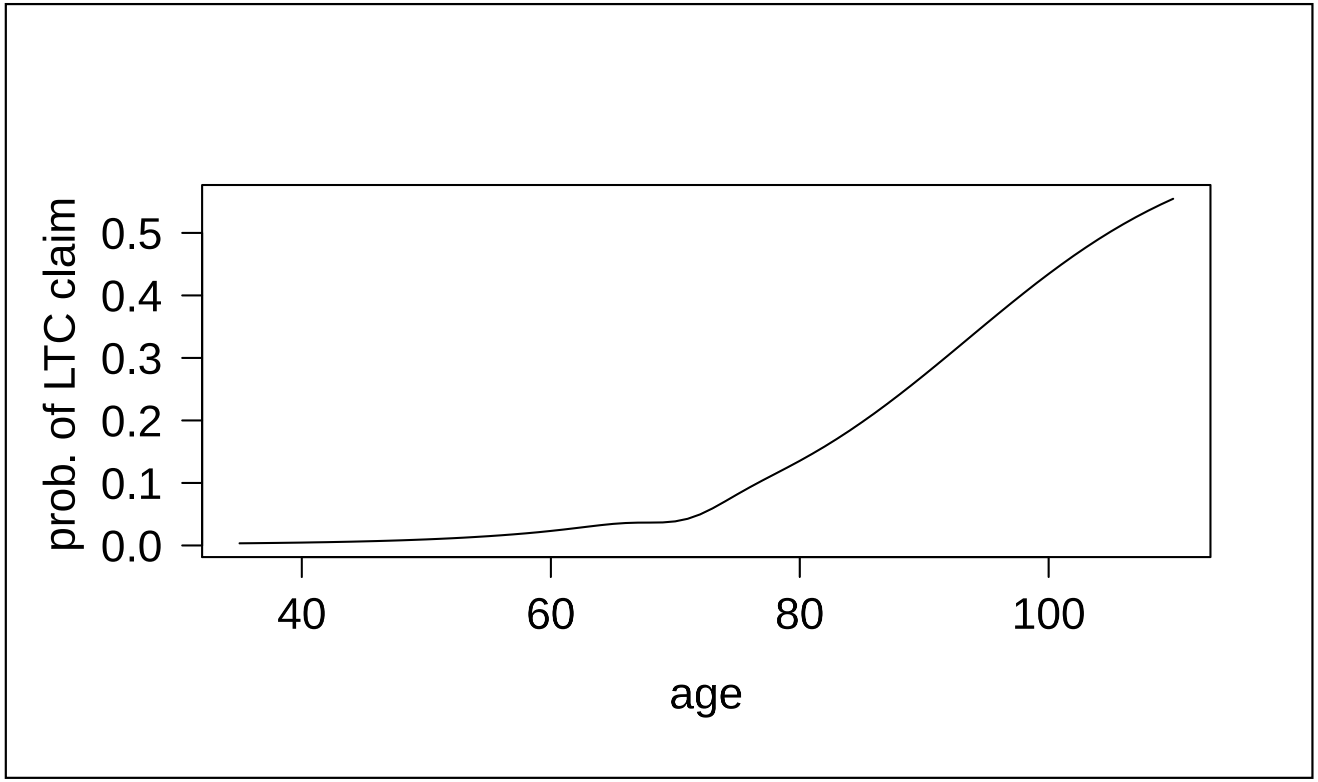
Assumption by Rickayzen and Walsh [2002]

$$w_x = \begin{cases} A + \dfrac{D-A}{1+B^{C-x}} & \text{for females} \\[2ex] \left(A + \dfrac{D-A}{1+B^{C-x}}\right)\left(1 - \dfrac{1}{3}\exp\left(-\left(\dfrac{x-E}{4}\right)^2\right)\right) & \text{for males} \end{cases}$$

Parameter	Females	Males
A	0.0017	0.0017
B	1.0934	1.1063
C	103.6000	93.5111
D	0.9567	0.6591
E	n.a.	70.3002

Parameters Rickayzen-Walsh

LTCI premiums: sensitivity analysis *(cont'd)*



Probability of disablement (Males)

Extra-mortality

Assumption by Rickayzen and Walsh [2002]

$$q_x^{i^{(k)}} = q_x^{[\text{standard}]} + \Delta(x, \alpha, k)$$

with:

$$\Delta(x, \alpha, k) = \frac{\alpha}{1 + 1.1^{50-x}} \, \frac{\max\{k-5, 0\}}{5}$$

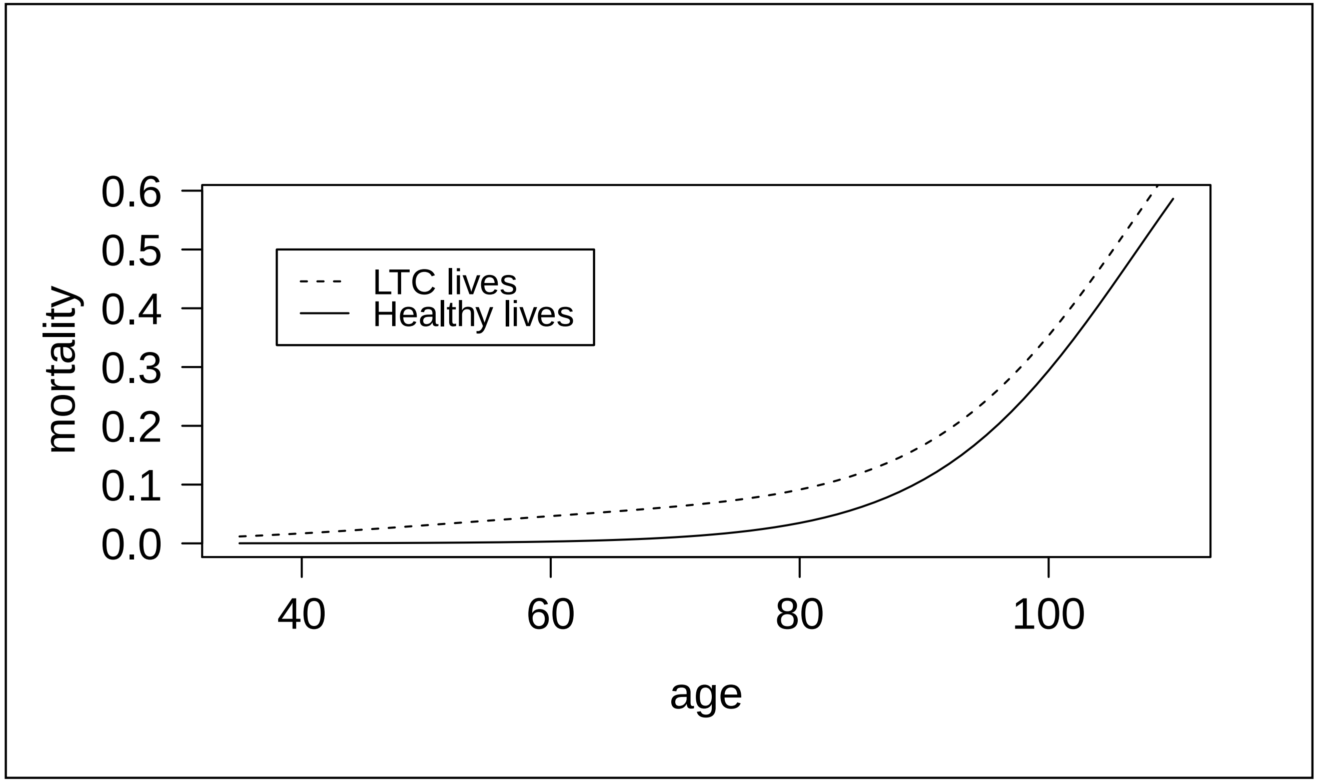
where:

- parameter k expresses LTC severity category
  - ▹ $0 \leq k \leq 5 \Rightarrow$ less severe $\Rightarrow$ no impact on mortality
  - ▹ $6 \leq k \leq 10 \Rightarrow$ more severe $\Rightarrow$ extra-mortality
- parameter α (assumption by Rickayzen [2007])

$\alpha = 0.10$ if $q_x^{[\text{standard}]} = q_x^{aa}$ (mortality of insured healthy people)

Our (base) choice: $\alpha = 0.10$, $k = 8$; hence:

$$q_x^i = q_x^{aa} + \Delta(x, 0.10, 8) = q_x^{aa} + \frac{0.06}{1 + 1.1^{50-x}}$$



Mortality assumptions (Males)

SENSITIVITY ANALYSIS

Sensitivity analysis concerning:

- probability of disablement (i.e. entering into LTC state)
- extra-mortality of lives in LTC state

Notation:

$\Pi_x^{[\mathrm{PX}]}(\delta, \lambda)$ = actuarial value (single premium) of product PX, according to the following assumptions:

- $\delta \;\Rightarrow\;$ disablement

  $$\bar{w}_x(\delta) = \delta\, w_x$$

  where w_x is given by the previous Eq.

- $\lambda \;\Rightarrow\;$ extra-mortality

  $$\bar{\Delta}(x;\lambda) = \lambda\, \Delta(x,\alpha,k) = \Delta(x, \lambda\, 0.10, 8)$$

  and hence:

  $$q_x^i(\lambda) = q_x^{aa} + \bar{\Delta}(x;\lambda)$$

For products P1, P2, P3, normalize and define the ratio:

$$\rho_x^{[\mathrm{PX}]}(\delta, \lambda) = \frac{\mathit{\Pi}_x^{[\mathrm{PX}]}(\delta, \lambda)}{\mathit{\Pi}_x^{[\mathrm{PX}]}(1, 1)}$$

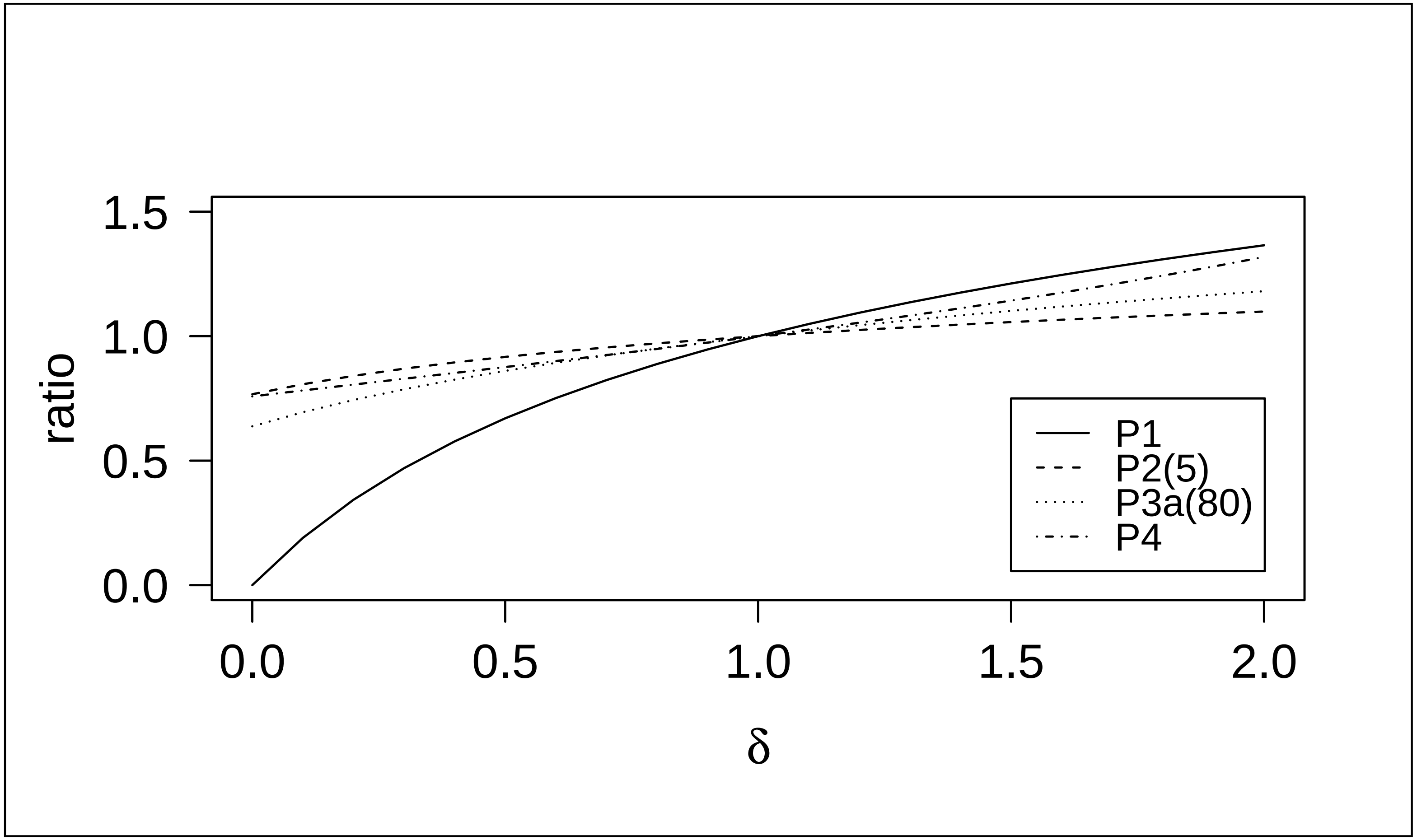
For product P4, with given b and b'', normalize and define the ratio:

$$\rho_x^{[\mathrm{P4}]}(\delta, \lambda) = \frac{b'(1, 1)}{b'(\delta, \lambda)}$$

For all the products, we first perform *marginal* analysis, i.e. tabulating the functions:

$$\mathit{\Pi}_x^{[\mathrm{PX}]}(\delta, 1),\ \rho_x^{[\mathrm{PX}]}(\delta, 1); \quad \mathit{\Pi}_x^{[\mathrm{PX}]}(1, \lambda),\ \rho_x^{[\mathrm{PX}]}(1, \lambda)$$

Sensitivity analysis: disablement assumption (parameter δ)



LTCI premiums: sensitivity analysis *(cont'd)*

δ	$\Pi_{50}^{[\mathrm{P1}]}(\delta, 1)$	$\rho_{50}^{[\mathrm{P1}]}(\delta, 1)$
0.0	0.00000	0.0000000
0.1	97.44457	0.1897494
0.2	176.07799	0.3428686
0.3	241.25240	0.4697798
0.4	296.47515	0.5773125
0.5	344.12555	0.6700999
0.6	385.86840	0.7513839
0.7	422.90118	0.8234961
0.8	456.10675	0.8881558
0.9	486.15044	0.9466585
1.0	513.54361	1.0000000
1.1	538.68628	1.0489592
1.2	561.89632	1.0941550
1.3	583.42997	1.1360865
1.4	603.49644	1.1751610
1.5	622.26854	1.2117151
1.6	639.89052	1.2460296
1.7	656.48397	1.2783412
1.8	672.15229	1.3088514
1.9	686.98406	1.3377327
2.0	701.05581	1.3651339

Product P1 (Stand-alone); $x = 50, b = 100$

LTCI premiums: sensitivity analysis *(cont'd)*

δ	$\Pi_{50}^{[P2(1)]}(\delta, 1)$	$\rho_{50}^{[P2(1)]}(\delta, 1)$	$\Pi_{50}^{[P2(5)]}(\delta, 1)$	$\rho_{50}^{[P2(5)]}(\delta, 1)$
0.0	492.1453	0.7446436	492.1453	0.7668209
0.1	522.4302	0.7904664	517.9195	0.8069802
0.2	547.3508	0.8281727	539.5114	0.8406230
0.3	568.3981	0.8600184	558.0108	0.8694472
0.4	586.5416	0.8874705	574.1426	0.8945825
0.5	602.4415	0.9115280	588.4118	0.9168156
0.6	616.5641	0.9328964	601.1825	0.9367139
0.7	629.2483	0.9520882	612.7241	0.9546971
0.8	640.7467	0.9694859	623.2411	0.9710837
0.9	651.2520	0.9853810	632.8914	0.9861200
1.0	660.9139	1.0000000	641.7995	1.0000000
1.1	669.8509	1.0135223	650.0652	1.0128789
1.2	678.1584	1.0260919	657.7693	1.0248828
1.3	685.9139	1.0378264	664.9783	1.0361152
1.4	693.1814	1.0488226	671.7475	1.0466625
1.5	700.0145	1.0591615	678.1234	1.0565969
1.6	706.4581	1.0689111	684.1455	1.0659801
1.7	712.5507	1.0781294	689.8475	1.0748645
1.8	718.3251	1.0868664	695.2586	1.0832956
1.9	723.8097	1.0951649	700.4040	1.0913127
2.0	729.0293	1.1030626	705.3059	1.0989504

Product P2 (Acceleration benefit); $x = 50, C = 1\,000$

LTCI premiums: sensitivity analysis *(cont'd)*

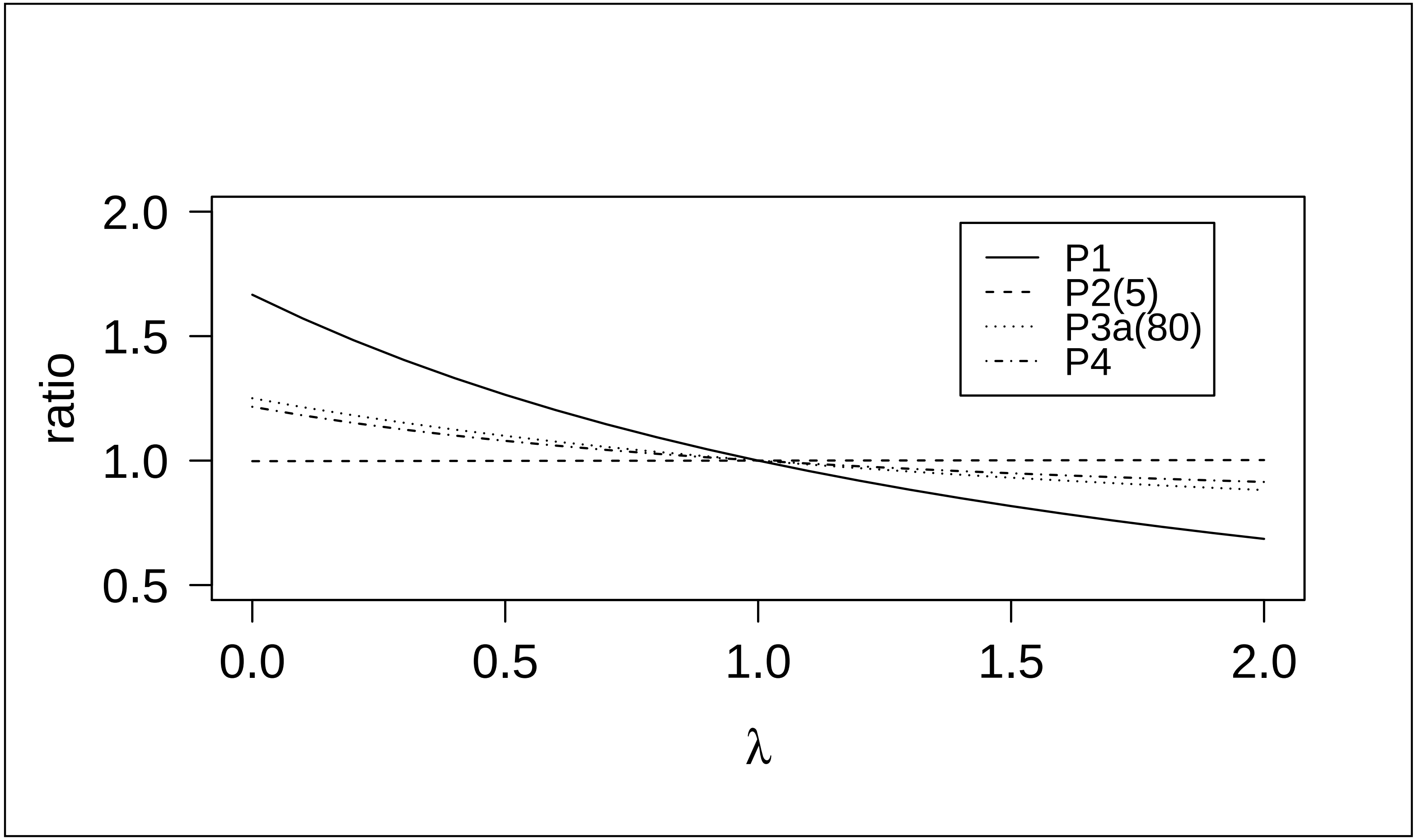
δ	$\Pi_{50}^{[\text{P3a}(80)]}(\delta, 1)$	$\rho_{50}^{[\text{P3a}(80)]}(\delta, 1)$	$\Pi_{50}^{[\text{P3b}(80)]}(\delta, 1)$	$\rho_{50}^{[\text{P3b}(80)]}(\delta, 1)$
0.0	700.5211	0.6379255	524.3054	0.6681005
0.1	762.7792	0.6946205	564.2116	0.7189513
0.2	816.5343	0.7435723	598.8261	0.7630591
0.3	863.9507	0.7867518	629.5434	0.8022009
0.4	906.4564	0.8254594	657.2615	0.8375209
0.5	945.0332	0.8605891	682.5844	0.8697888
0.6	980.3808	0.8927781	705.9351	0.8995436
0.7	1 013.0142	0.9224956	727.6214	0.9271776
0.8	1 043.3239	0.9500969	747.8754	0.9529864
0.9	1 071.6132	0.9758584	766.8772	0.9771996
1.0	1 098.1236	1.0000000	784.7703	1.0000000
1.1	1 123.0514	1.0227003	801.6718	1.0215369
1.2	1 146.5586	1.0441071	817.6790	1.0419342
1.3	1 168.7817	1.0643443	832.8740	1.0612966
1.4	1 189.8365	1.0835178	847.3271	1.0797136
1.5	1 209.8231	1.1017185	861.0993	1.0972629
1.6	1 228.8288	1.1190259	874.2436	1.1140122
1.7	1 246.9299	1.1355096	886.8072	1.1300214
1.8	1 264.1943	1.1512313	898.8317	1.1453438
1.9	1 280.6825	1.1662462	910.3545	1.1600268
2.0	1 296.4487	1.1806036	921.4091	1.1741132

Products P3a and P3b (Insurance packages); $x = 50, C = 1\,000, b' = 50, b'' = 150$

δ	$b'(\delta,1)$	$\rho_x^{[P4]}(\delta,1)$
0.0	100.00000	0.7582433
0.1	96.96404	0.7819840
0.2	94.13166	0.8055136
0.3	91.47026	0.8289506
0.4	88.95221	0.8524165
0.5	86.55461	0.8760288
0.6	84.25873	0.8998988
0.7	82.04926	0.9241317
0.8	79.91365	0.9488283
0.9	77.84153	0.9740858
1.0	75.82433	1.0000000
1.1	73.85486	1.0266668
1.2	71.92708	1.0541833
1.3	70.03587	1.0826500
1.4	68.17685	1.1121713
1.5	66.34626	1.1428576
1.6	64.54086	1.1748267
1.7	62.75783	1.2082052
1.8	60.99468	1.2431301
1.9	59.24927	1.2797513
2.0	57.51967	1.3182330

Product P4 (Enhanced pension); $x = 65$, $b = 100$, $b'' = 150$

Sensitivity analysis: extra-mortality assumption (parameter λ)



LTCI premiums: sensitivity analysis *(cont'd)*

λ	$\Pi_{50}^{[\mathrm{P1}]}(1,\lambda)$	$\rho_{50}^{[\mathrm{P1}]}(1,\lambda)$
0.0	855.7094	1.6662838
0.1	806.6737	1.5707987
0.2	761.9567	1.4837234
0.3	721.0856	1.4041370
0.4	683.6467	1.3312339
0.5	649.2769	1.2643073
0.6	617.6576	1.2027364
0.7	588.5080	1.1459748
0.8	561.5807	1.0935405
0.9	536.6571	1.0450079
1.0	513.5436	1.0000000
1.1	492.0686	0.9581828
1.2	472.0797	0.9192592
1.3	453.4411	0.8829652
1.4	436.0319	0.8490650
1.5	419.7439	0.8173482
1.6	404.4804	0.7876263
1.7	390.1547	0.7597305
1.8	376.6889	0.7335090
1.9	364.0128	0.7088255
2.0	352.0634	0.6855570

Product P1 (Stand-alone); $x = 50, b = 100$

LTCI premiums: sensitivity analysis *(cont'd)*

λ	$\Pi_{50}^{[P2(1)]}(1,\lambda)$	$\rho_{50}^{[P2(1)]}(1,\lambda)$	$\Pi_{50}^{[P2(5)]}(1,\lambda)$	$\rho_{50}^{[P2(5)]}(1,\lambda)$
0.0	660.9139	1	640.3371	0.9977214
0.1	660.9139	1	640.4879	0.9979563
0.2	660.9139	1	640.6376	0.9981896
0.3	660.9139	1	640.7863	0.9984213
0.4	660.9139	1	640.9341	0.9986515
0.5	660.9139	1	641.0808	0.9988801
0.6	660.9139	1	641.2265	0.9991071
0.7	660.9139	1	641.3712	0.9993326
0.8	660.9139	1	641.5150	0.9995566
0.9	660.9139	1	641.6577	0.9997791
1.0	660.9139	1	641.7995	1.0000000
1.1	660.9139	1	641.9404	1.0002194
1.2	660.9139	1	642.0802	1.0004374
1.3	660.9139	1	642.2191	1.0006538
1.4	660.9139	1	642.3571	1.0008688
1.5	660.9139	1	642.4941	1.0010822
1.6	660.9139	1	642.6302	1.0012943
1.7	660.9139	1	642.7653	1.0015048
1.8	660.9139	1	642.8995	1.0017140
1.9	660.9139	1	643.0328	1.0019216
2.0	660.9139	1	643.1652	1.0021279

Product P2 (Acceleration benefit); $x = 50, C = 1\,000$

LTCI premiums: sensitivity analysis *(cont'd)*

λ	$\Pi_{50}^{[\mathrm{P3a(80)}]}(1,\lambda)$	$\rho_{50}^{[\mathrm{P3a(80)}]}(1,\lambda)$	$\Pi_{50}^{[\mathrm{P3b(80)}]}(1,\lambda)$	$\rho_{50}^{[\mathrm{P3b(80)}]}(1,\lambda)$
0.0	1 373.1426	1.2504444	1 030.1514	1.3126789
0.1	1 333.7360	1.2145591	992.0364	1.2641106
0.2	1 297.7979	1.1818323	957.9426	1.2206663
0.3	1 264.9490	1.1519186	927.4057	1.1817544
0.4	1 234.8573	1.1245157	900.0200	1.1468579
0.5	1 207.2314	1.0993584	875.4306	1.1155246
0.6	1 181.8156	1.0762136	853.3264	1.0873583
0.7	1 158.3843	1.0548760	833.4345	1.0620108
0.8	1 136.7389	1.0351648	815.5147	1.0391763
0.9	1 116.7039	1.0169200	799.3555	1.0185853
1.0	1 098.1236	1.0000000	784.7703	1.0000000
1.1	1 080.8603	0.9842793	771.5943	0.9832104
1.2	1 064.7915	0.9696463	759.6816	0.9680305
1.3	1 049.8081	0.9560017	748.9029	0.9542957
1.4	1 035.8128	0.9432570	739.1434	0.9418596
1.5	1 022.7189	0.9313331	730.3010	0.9305921
1.6	1 010.4485	0.9201591	722.2849	0.9203775
1.7	998.9319	0.9096716	715.0140	0.9111125
1.8	988.1065	0.8998136	708.4160	0.9027050
1.9	977.9161	0.8905337	702.4263	0.8950725
2.0	968.3098	0.8817858	696.9867	0.8881411

Products P3a and P3b (Insurance packages); $x = 50, C = 1\,000, b' = 50, b'' = 150$

LTCI premiums: sensitivity analysis *(cont'd)*

λ	$b'(1,\lambda)$	$\rho_x^{[\mathrm{P4}]}(1,\lambda)$
0.0	62.34898	1.2161277
0.1	64.17119	1.1815946
0.2	65.86125	1.1512738
0.3	67.43103	1.1244723
0.4	68.89119	1.1006390
0.5	70.25128	1.0793302
0.6	71.51992	1.0601847
0.7	72.70488	1.0429056
0.8	73.81315	1.0272469
0.9	74.85106	1.0130027
1.0	75.82433	1.0000000
1.1	76.73813	0.9880920
1.2	77.59716	0.9771534
1.3	78.40567	0.9670771
1.4	79.16755	0.9577704
1.5	79.88630	0.9491531
1.6	80.56513	0.9411556
1.7	81.20698	0.9337169
1.8	81.81451	0.9267834
1.9	82.39015	0.9203081
2.0	82.93615	0.9142494

Product P4 (Enhanced pension); $x = 65$, $b = 100$, $b'' = 150$

Joint sensitivity analysis (parameters δ, λ)

For the generic product PX, and a given age x, find (δ, λ) such that:

$$\rho_x^{[\mathrm{PX}]}(\delta, \lambda) = \rho_x^{[\mathrm{PX}]}(1, 1) = 1 \qquad (*)$$

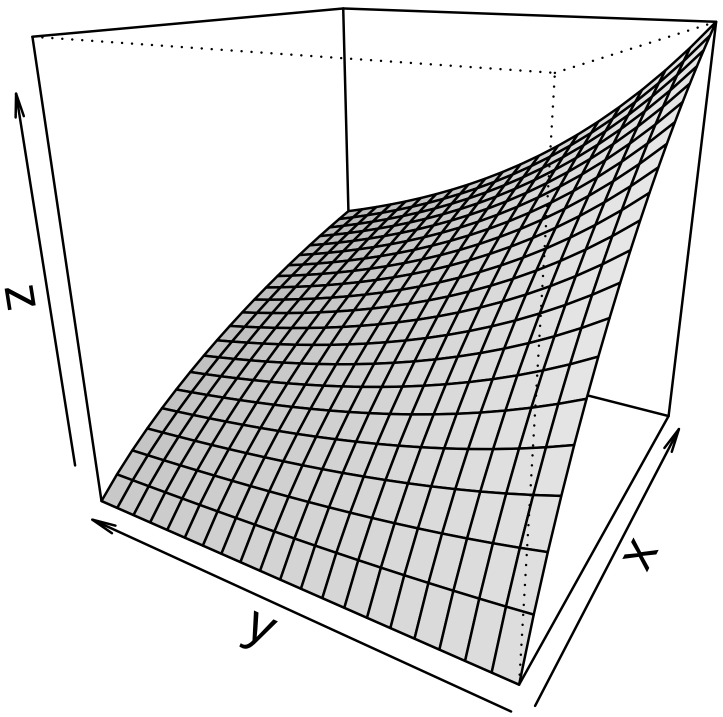
Eq. $(*)$ implies

- for products P1, P2, P3:

$$\mathit{\Pi}_x^{[\mathrm{PX}]}(\delta, \lambda) = \mathit{\Pi}_x^{[\mathrm{PX}]}(1, 1)$$

- for product P4:

$$b'(\delta, \lambda) = b'(1, 1)$$

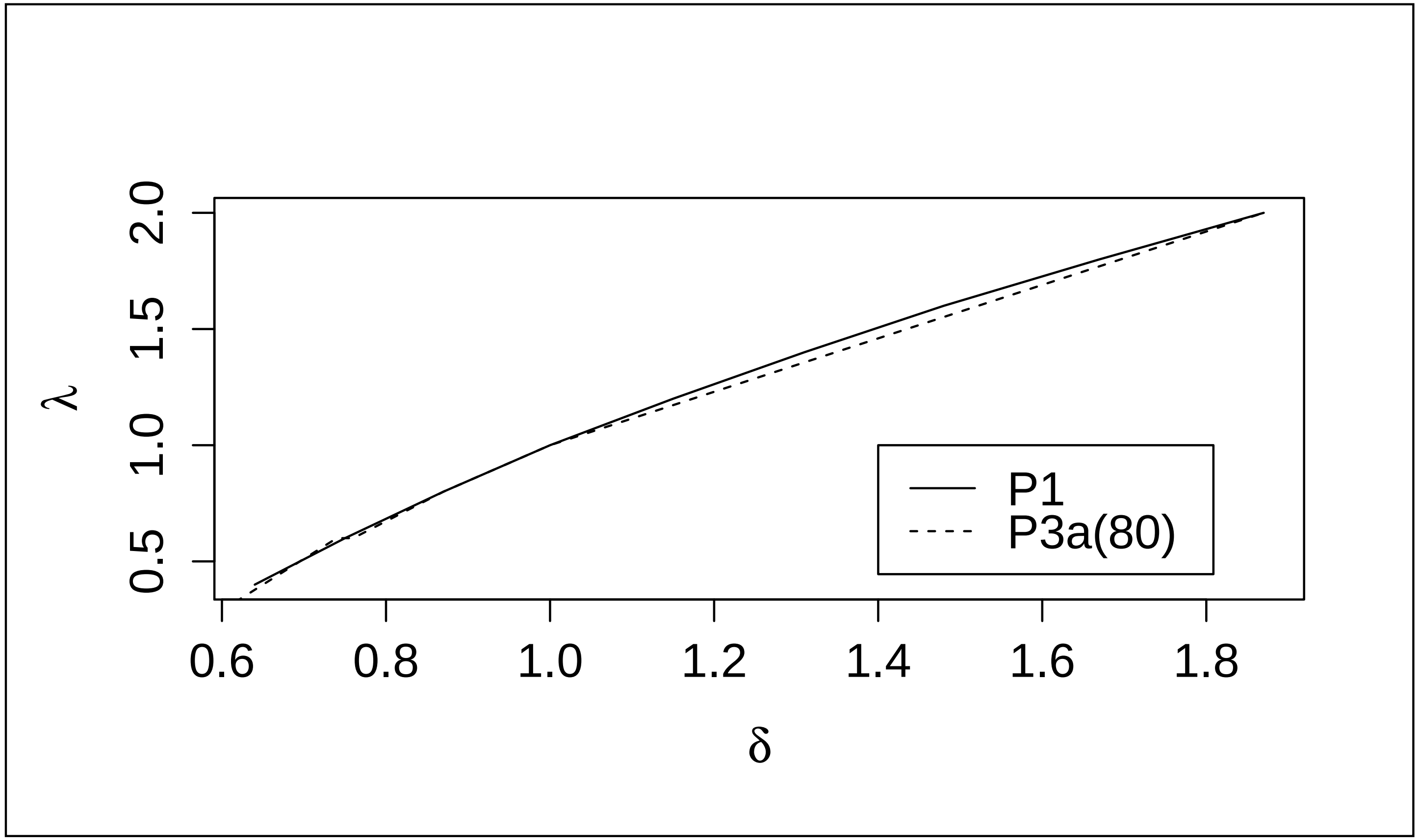


Product P3a(80)

x = δ $\Rightarrow$ *disablement*

y = λ $\Rightarrow$ *extra-mortality*

z = Π $\Rightarrow$ *premium*



Offset effect: isopremium lines

CONCLUDING REMARKS

Combined LTCI products: mainly aiming at reducing the relative weight of the risk component by introducing a "saving" component, or by adding the LTC benefits to an insurance product with an important saving component

Combined insurance products in the area of health insurance:

- Insurer's perspective
  - ▷ a combined product can result profitable even if one of its components is not profitable
  - ▷ a combined product can be less risky than one of its components (less exposed to impact of uncertainty risk related to the choice of technical bases)
- Client's perspective $\Rightarrow$ purchasing a combined product can be less expensive than separately purchasing all the single components (in particular: reduction of acquisition costs charged to the policyholder)

LTCI premiums: sensitivity analysis *(cont'd)*

Examples are provided by:

- LTC covers as riders to life insurance; see:
  - ▷ acceleration benefit in whole life assurance
  - ▷ LTC annuity in enhanced pension
- LTC covers in insurance packages; see:
  - ▷ packages including old-age deferred life annuity and death benefit